ADVANCED ALGEBRA
WITH FINANCIAL APPLICATIONS
Using
Financial Algebra 2e (NGL/Cengage Learning)

COURSE DOCUMENT

(A)
COURSE DESCRIPTION

Advanced Algebra with Financial Applications is a mathematical modeling course that is algebra-based, applications-oriented, and technology-dependent. The course addresses college preparatory mathematics topics from Algebra 2, Statistics, Probability, Precalculus, and Calculus under eight financial umbrellas: Discretionary Expenses, Banking, Investing, Credit, Employment and Income Taxes, Automobile Ownership, Independent Living, and Retirement Planning and Household Budgeting. The course allows students to experience the interrelatedness of mathematical topics, find patterns, make conjectures, and extrapolate from known situations to unknown situations. The mathematics topics contained in this course are introduced, developed, and applied in an as-needed format in the financial settings covered. Students are encouraged to use a variety of problem-solving skills and strategies in real-world contexts, and to question outcomes using mathematical analysis and data to support their findings. The course offers students multiple opportunities to use, construct, question, model, and interpret financial situations through symbolic algebraic representations, graphical representations, geometric representations, and verbal representations. It provides students a motivating, young-adult centered financial context for understanding and applying the mathematics they are guaranteed to use in the future.

(B)
COURSE PURPOSE, RATIONALE AND JUSTIFICATION
WHAT STUDENT NEEDS ARE ADDRESSED?

Advanced Algebra with Financial Applications offers 11th and 12th grade students an opportunity to view the world of finance through a mathematical lens. The mathematical formulas, functions, and pictorial representations used assist students in making sense of the financial world around them and equip them with the ability to make sound financial decisions. The overarching purpose of the course is to develop mathematically proficient students who can apply mathematics to solve real world problems that arise at home, at school, in society, and in the workplace.

The Advanced Algebra with Financial Applications course is a vehicle by which students can learn Algebra 2 and Precalculus concepts that will apply to their daily lives. Thus, these more advanced concepts are taught with an intrinsically motivating backdrop, which raises student engagement. This maximizes the chances for success for students who might not have succeeded in a traditional Algebra 2 or Precalculus course and gives them a chance to develop strength and confidence in their mathematics ability.

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COURSE OUTLINE

Unit 1: Discretionary Expenses

In this unit, students will use statistics to describe trends in their discretionary expenses. They will compute measures of central tendency (mean, median and mode) and measures of dispersion (range, mean deviation, absolute mean deviation, variance and standard deviation) of frequency distributions and learn the advantages and disadvantage of each. Students will interpret areas under the normal curve as relative frequencies, compute z-scores given raw scores, compute raw scores given z-scores, and read and interpret the normal curve areas table. Students will plot and interpret bivariate data through the use of scatterplots and linear regression equations. Students will compute the Pearson Product-Moment Coefficient of Correlation and based on the circumstances decide whether causation is an explanation for the correlation.

Common Core State Standards for Mathematical Content addressed:
S-DI1, S-ID2, S-ID3, S-ID4, S-ID5, S-ID6, S-ID7, S-ID8, S-ID
Common Core Standards for Mathematical Practice addressed: MP1, MP2, MP4, MP5, MP6, MP7, MP8.

Mathematics Learning Goals Cross-Referenced with Textbook and Projects

- **Students will learn measures of central tendency.** Students will compute mean, median and mode of frequency distributions and learn the advantages and disadvantage of each. *Financial Algebra 2E* sections 1-1 and 1-2 introduce these concepts and they are spiraled throughout the rest of the units.
- **Students will learn how to use and interpret Sigma notation.** Students will be offered a variety of opportunities to express the mean in terms of sigma notation and to interpret equations that employ sigma notation. *Financial Algebra 2E* section 1-1 offers a variety of expense-based situations where determination of the mean assists in understanding the data.
- **Students will transform raw data into a frequency distribution**. In *Financial Algebra 2E* sections 1-1 and 1-2, students chart spending and expense data using frequency, relative frequency, cumulative frequency, and relative cumulative frequency charts. Project 1.1 asks students to chart Internet automobile data and create frequency distributions for that data.
- **Students will scaffold the measures of dispersion to understand the derivation of the standard deviation.** The range, mean deviation, absolute mean deviation, variance and standard deviation will be derived and computed. Advantages and disadvantages of each measure will be analyzed. *Financial Algebra 2E* section 1-3 introduces these concepts and they are spiraled throughout the rest of the units.
- **Students will create and interpret frequency distributions and understand weighted statistics.** In *Financial Algebra 2E* section 1-2 students will see how frequencies affect measures of central tendency and dispersion.
- **Students will compute relative frequencies and compare them to raw frequencies.** In *Financial Algebra 2E* section 1-2, the advantages and disadvantages of relative frequencies will be scrutinized. Project 1.4 has students performing an experiment to predict frequencies using experimentally-generated relative frequencies.
- **Students will plot and interpret bivariate data.** This will include scatterplots, regression lines, and correlation coefficients. Slopes of regression lines will be interpreted as rates of change. *Financial Algebra 2E* section 1-5 introduces bivariate data, scatterplots, correlation coefficient and the slope of the regression line. Regression is revisited often in the rest of the units. Project 1.5 will have students gathering spending data and using regression analysis to find trends and make predictions based on that data.
- **Students will construct and interpret scatterplots**. *Financial Algebra 2E* section 1-5 introduces students to scatter plots as an important part of regression analysis. Students will identify form, direction, and strength from a scatterplot.
- **Students will delineate causation vs. correlation for bivariate data.** In *Financial Algebra 2E* section 1-5, students examine the concepts of causal and correlational relationships in the context of discretionary and essential spending.
- **Students will find, interpret, and graph linear regression equations and determine the correlation coefficient.** *Financial Algebra 2E* section 1-5, students determine domains for which prediction using a regression line is considered extrapolation or interpolation as they model spending data. Students find and interpret the Pearson Product-Moment Coefficient of Correlation and use that value to make inferences about the line of best fit.
- **Students will interpret findings using the Normal Curve.** In *Financial Algebra 2E* section 1-4 students will interpret areas under the normal curve as relative frequencies,

and compute z-scores given raw scores. They will also compute raw scores given z-scores. They will read the normal curve areas table and draw bell curves and shade in appropriate intervals under the curve. The Appendix has the Normal Curve areas table. In Project 1.2 students will analyze heights of basketball players to see if they are normally distributed.

- **Students will understand the relationships in the Normal Curve based on standard deviations.** In *Financial Algebra 2E* section 1-4 students will interpret the likelihood of scores at different standard deviations away from the mean. Students will interpret z-scores as units of standard deviations from the mean.
- **Students will find raw scores that correspond to given percentiles of Normal distributions.** Problems requiring students to find thresholds for the top 10%, top 5%, etc., are analyzed in *Financial Algebra Second Edition* section 1-4.

Unit 2: Banking Services

In this unit, students will use the simple and compound interest formulas and use inverse operations including logarithms and logarithmic operations to solve for missing variables in interest problems. In preparation for deriving the natural base e, students will analyze rational function behavior and limits of rational functions as the independent variable approaches infinity. With that as a basis, students will approximate the natural base e by examining the exponential sequence for increasing values of the exponent and denominator both with and without limit notation, and thusly derive and apply the natural base e in the continuous compounding formula. Students will be able to identify/use exponential equations as models for exponential decay when the exponent is less than 1 and identify/use exponential equations as models for exponential growth when the exponent is greater than 1. They will use arithmetic and geometric sequences and series to model periodic savings plans.

Common Core State Standards for Mathematical Content addressed:
A-CED4, A-SSE1a, A-SSE1b, A-SSE3, F-IF4, F-IF8b, F-BF1a, 2, 5, N-RN1, N-RN2
Common Core Standards for Mathematical Practice addressed: MP1, MP4, MP5, MP6, and MP8

Mathematics Learning Goals Cross-Referenced with Textbook and Projects

- **Students will use the simple interest formula and use inverse operations to solve for missing variables in interest problems.** In *Financial Algebra 2E* section 2-3, students learn how to compute I (interest), P (principal), R (rate), and T (time). They solve for one of the variables in terms of the others in numeric and literal equations. They use number sense to confirm the reasonableness of their answers.
- **Students will interpret periodic savings plans without interest as arithmetic sequences.** *Financial Algebra Second Edition* section 2-3 shows the common ratio and formulas to find the n^{th} term of an arithmetic sequence.
- **Students will use iteration to show how compounding pays "interest on your interest."** *Financial Algebra 2E* section 2-4 shows iterative processes that underscore the meaning of compound interest. Students use calendars to document the iteration and see where interest is added and then used as principle to compute subsequent interest. Project

2.1 requires students to guess future values based on the intuitive notion of compound interest.

- **Students will derive the compound interest formula by using patterns and inductive reasoning.** *Financial Algebra 2E* section 2-4 algebraically derives the formula for quarterly compounding, and then uses inductive reasoning to extend the formula into compounding over any period of time. Project 2.2 involves looking for patterns to make inductive inferences about a generalized compound interest formula.
- **Students will compute compound interest with and without the formula.** *Financial Algebra 2E* sections 2-3 through 2-6 illustrate compounding with and without the formulas and shows how daily compounding is adjusted for deposits and withdrawals.
- **Students will apply and interpret limit notation.** *Financial Algebra 2E* section 2-6 investigates limits since they will be used in the concept of continuous compounding. Students will compute limits of rational functions and other expressions. Project 2.3 will highlight a limit that is central to the continuous compounding formula.
- **Students will model an infinite geometric series and find a finite sum for an infinite series with common ratio ½**. *Financial Algebra 2E* section 2-6 features the classic problem where someone walks across a room by first walking half the room, then half of the remaining half, etc. Students realize that they do actually cover the entire room, and that the sum is 1.
- **Students will compute limits of rational functions as x approaches infinity.** *Financial Algebra 2E* section 2-6 has students examining numerators and denominators to intuitively determine function values as variable quantities approach infinity.
- **Students will approximate the natural base *e* by examining the exponential sequence for increasing values of the exponent and denominator**. *Financial Algebra 2E* section 2-6 discuss the "battle" between the decreasing base and the increasing exponent, which ends up with the natural base e. Project 2.3 uses calculator tables to make inferences about a rational function's pattern as a variable approaches infinity.
- **Students will inductively derive the natural base *e* using limits.** *Financial Algebra 2E* section 2-6 has an activity that displays the pattern leading to the natural base e. This is a component of Project 2.3.
- **Students will apply the natural base *e* in the formula for continuous compounding of interest.** *Financial Algebra 2E* section 2-6 uses the formula for continuous compounding to model the effect as compared to other types of compounding.
- **Students will be able to identify equations as models for exponential decay when the exponent is less than 1**. *Financial Algebra 2E* section 4-6 models exponential decay by looking at depreciation of an automobile which offers a contrast to growth via compound interest.
- **Students will be able to identify equations as models for exponential growth when the exponent is greater than 1.** *Financial Algebra 2E* section 4-6 uses exponential growth to model rent increases. The constant percent of annual increase is modeled by an exponential function.
- **Students will graph exponential functions.** *Financial Algebra 2E* section 4-6 has students using exponential regression on the calculator to model exponential functions. These functions are used to model automobile prices over the years.
- **Students will analyze rational function behavior and limits of rational functions as the independent variable approaches infinity**. *Financial Algebra 2E* section 2-6 has

students examining numerators and denominators to compare their growth. The relative size of the numerators and denominators, and the powers of the highest exponents lead students to find rules for rational function limits.

- **Students will compute Annual Percentage Yield (APY), given the Annual Percentage Rate (APR).** *Financial Algebra 2E* section 2-5 derives formulas used to compute and compare APY and APR.
- **Students will use the compound interest formula to derive the rational function that models the present value of a single deposit investment formula**. *Financial Algebra 2E* section 2-8 derives the present value formula in the context of compound interest.
- **Students will use the compound interest formula to derive the rational function that models the present value of a periodic deposit investment**. *Financial Algebra 2E* section 2-8 adjusts the present value formula to include periodic deposits.
- **Students will use the future value of a periodic deposit investment formula**. *Financial Algebra 2E* section 2-7 uses the formulas for the future value of a single-deposit investment and for a periodic deposit. Project 2.4 applies this formula to college costs, a concern of many high school and college students. They also have to imagine they are young parents and have to plan to save for college costs in the future, which they must compute using regression.
- **Students will adapt the algebra from banking formulas for input into a spreadsheet**. *Financial Algebra 2E* sections 2-4 and 2-7 require students to adapt the structure of algebraic thinking to the language understood by the spreadsheet program. This requires intense attention to precision since any deviation from the correct syntax will prevent the spreadsheet from computing correctly.
- **Students will explore logarithmic notation and interpret logarithms as exponents**. In *Financial Algebra 2E* sections 2-9 and 2-10, students will be solving some of the banking formulas for the exponent, requiring some basic logarithm rules.
- **Students will alter logarithmic expressions using the one-to-one and power properties.** In *Financial Algebra 2E* sections 2-9 and 2-10, students will use 2 properties of logarithms to make simplifications which will be viewed in the light of the laws of exponents, to further underscore the notion that logarithms are just exponents. Project 2.5 asks students to use logarithms to determine the term necessary for a savings plan.

Unit 3: Investing

Students are introduced to basic business organization terminology in order to read, interpret, chart and algebraically model stock ownership and transaction data. They use algebraic ratios and proportions to model percent increases, decreases, moving averages, stock splits and dividend yields. Different broker fee schedules are compared for different stock transactions. Signed numbers, candlestick charts and line graphs are used to statistically monitor trends in prices.

Students also learn how entrepreneurs use randomized designs, matched-pair designs, observational studies, hypothesis testing, and inferential statistics to make decisions in the development of new businesses and products. In determining the efficacy of producing a product, students will learn how to create and interpret supply and demand curves, expense equations, revenue equations, profit equations and paired systems of these equations as critical components of breakeven analyses. Since the revenue and profit equations are both quadratic, the analyses will

entail the use and interpretation of the axis of symmetry, vertex, roots (both real and complex), concavity, intersection points, relative extrema, absolute extrema, and domains in the context of the problem situation. The unit culminates with the study of finding optimal outcomes through the use of linear programming techniques.

Common Core State Standards for Mathematical Content that are Addressed: A-CED1, A-CED2, A-CED3, A-CED4 , A-REI2, A-REI3, A-REI4b, A-REI6, A-REI7, A-REI10, A, REI11,A-REI12, A-SSE1, F-IE4 , F-IF1, F-IF4, F-IF5, F-IF7a, F-IF8, S-ID6, S-ID8, N-Q1, N-Q2, N-Q3, N-CN , S-ID8, S-ID9, S-IC1, S-IC3, S-IC5, S-IC6
Common Core Standards for Mathematical Practice addressed: MP1, MP2, MP3, MP4, and MP5.

Mathematical Learning Goals Cross-Referenced with Textbook and Projects

- **Students will construct, use, and interpret algebraic ratios and proportions**. *Financial Algebra 2E* section 8-1 examines ratios as used to model business partnership shares. Here, given a set of n compound ratios and a total T students write and solve equations in terms of n and T where the variable coefficients are the ratios and determine the amount associated with each ratio. *Financial Algebra 2E* sections 8-2, 8-6, 8-8, and 8-9 explore the use of ratio and proportions in the context of stock market transactions, stock prices, and stock dividends. *Financial Algebra 2E* sections 9-5, and 9-6 offer students opportunities to model and interpret ratios and proportions within the context of a new business venture. In all of these sections, students use, interpret and evaluate rational expressions, algebraic fractions, ratios, and proportions
- **Students will determine, use, and interpret percent increase/decrease of monetary amounts**. *Financial Algebra 2E* section 8-2 has students determine percent increase and decrease in the context of stock prices. Students then translate what they have learned to writing algebraic and spreadsheet formulas for percent net change.
- **Students will construct and interpret pictorial representations of data.** *Financial Algebra 2E* sections 8-3 and 8-4 teaches students how to chart and interpret stock market trends pictorially and graphically. *Financial Algebra 2E* Chapter 9 focuses on the mathematical modeling of a business. Here, students use regression analysis, functions, and graphs to model how expense, revenue and profit functions work together to assist in setting the per unit price that will result in the maximum profit. Project 3.1 applies these skills in the context of charting corporate stocks using OHLC and Candlestick charts. Students also chart multiple time intervals of simple moving averages for closing prices in order to analyze crossover graphs. Based upon the interpretations of these graphs, students are asked to identify and explain stock trend reversal points.
- **Given a set of *n* data points, students will calculate and interpret *d*-day simple moving averages by applying the Arithmetic Average Formula and the Subtraction/Addition Method.** *Financial Algebra 2E* section 8-4 introduces students to the concept of data smoothing through the use of simple moving averages. Students calculate, chart, and interpret the simple moving average crossover graphs to make informed decisions about stock trend reversals. Project 3.1 uses real world data collected by students to reinforce the smoothing effects that simple moving averages have on that data.

- **In any *a-for-b* stock split, where *P* represent the pre-split price per share, students will calculate the post-split price per share by setting up and calculating the product of the split ratio b/a and the pre-split price per share P.** *Financial Algebra 2E* section 8-8 employs ratio and proportions to create pre- and post-split market price functions.
- **In any *a-for-b* stock split, where *D* represent the pre-split number of shares, students will calculate the post-split number of shares using the split ratio a/b.** *Financial Algebra 2E* section 8-8 employs ratios and proportions to create pre- and post-split outstanding shares functions.
- **Students will calculate the stock yield percentage using the yield ratio A/C where A represents the annual dividend per share and C represents the current price per share.** *Financial Algebra 2E* section 8-9 defines yield as a ratio and offers students opportunities to calculate monetary yield amounts as well as write algebraic expressions that can be used to create yield formulas.
- **Students will create and evaluate functions and use them to model situations.** *Financial Algebra 2E* Chapters 8 and 9 have students creating, using, and graphing functions in the contexts of the stock market and business entrepreneurship.
- **Students will use regression analysis to predict the closing price of a stock.** In *Financial Algebra 2E* section 8-4, students are asked to examine closing price stock data for consecutive trading days. They use a scatterplot and determine the regression curve that best fits the data. Then, using that regression equation, they predict the closing price.
- **Students will translate verbal situations into algebraic linear functions.** *Financial Algebra 2E* sections 8-1, 8-2, 8-6, 8-7, 8-8, 8-9, and 8-10 offer students opportunities to model stock market situations using linear functions. Students also use linear functions in section 9-3 to model product demand and in section 9-4 to model fixed and variable expenses. Project 3.2 asks students to model expense and demand using linear functions.
- **Students will translate verbal situations into quadratic functions**. *Financial Algebra 2E* sections 9-5 and 9-6 explore the use of quadratic functions to model revenue and profit. Project 3.2 has students model revenue and profit using quadratic functions.
- **Students will translate verbal situations into linear and quadratic inequalities.** *Financial Algebra 2E* sections 9-6 through 9-8 use linear and quadratic inequalities to contextualize the relationships between and among expense, revenue, and profit functions.
- **Students will solve linear systems of equations and inequalities and identify points of intersection and domains in the context of the problem situation.** *Financial Algebra 2E* section 9-4 explores the use of linear systems to model revenue and expense situations. Students identify domains for which $f(x) > g(x)$, $f(x) = g(x)$, and $f(x) < g(x)$. Project 3.2 asks students to contextualize the domains of intersecting functions $f(x)$ and $g(x)$ where $f(x) > g(x)$, $f(x) = g(x)$, and $f(x) < g(x)$.
- **Students will find the axis of symmetry, vertex, roots, and the concavity of parabolic curves.** *Financial Algebra 2E* section 9-6 uses parabolas to model breakeven analysis. Students use the quadratic formula to find the roots of a quadratic equation and interpret those roots in the context of the problem situation. Project 3.2 asks students to contextualize the axis of symmetry, vertex, roots and concavity of quadratic revenue and profit functions.
- **Students will solve linear-quadratic systems of equations and inequalities, and interpret the roots, intersection points, relative extrema, absolute extrema, and**

domains in the context of the problem situation. *Financial Algebra 2E* sections 9-6 through 9-8 model business contexts using linear-quadratic systems of equations. Project 3.2 uses linear-quadratic systems of equalities and inequalities to contextualize the modeling of product price setting for optimal profit.

- **Students will use the transitive property of dependence.** *Financial Algebra 2E* section 9-8 sets the stage for the linear/quadratic business model by examining the concept of variable dependence (if x depends on y, and y depends on z, it follows that x depends on z.)
- **Students will learn to solve quadratic equations by completing the square.** The revenue and profit functions are concave down parabolas and the roots are very significant. In *Financial Algebra 2E* section 9-5, students can factor if possible, use the quadratic formula, or complete the square.
- **Students will determine the zero net difference.** *Financial Algebra 2E* section 9-6 defines zero net difference functions as a function that is the difference of a quadratic revenue function and a linear expense function. The zero net difference points are the points of intersection of the linear/quadratic system.
- **Students will write algebraic formulas for use in spreadsheets.** *Financial Algebra 2E* sections 8-2, 8-4, 8-8, 8-9, 9-2, and 9-6 ask students to create stock market and business model linear and quadratic formulas for use in spreadsheets.
- **Students will interpret imaginary roots in profits equations.** In *Financial Algebra 2E* section 9-7, students encounter complex routes in real-life business situations.
- **Students will employ linear programming to solve optimization problems.** *Financial Algebra 2E* section 9-8 introduces students to the concepts, methods and applications of linear programming. Graphs and inequalities will be created to determine feasible solutions and optimal solutions in a variety of financial contexts. Project 3.4 has students interviewing a financial advisor in order to determine an optimal investment strategy using linear programming.
- **Students will learn how to plan a completely randomized design and a matched pairs design.** Schematics for completely randomized designs will be drawn. Experiments will be evaluated to see if they can be done in a matched pairs design. Pros and cons of each design will be discussed. *Financial Algebra Second Edition* section 9-1 centers on bias, control, replication and randomization in statistical experimental designs.
- **Students will learn the difference between and experimental and observational study.** In *Financial Algebra 2E* section 9-1 students learn about the difference between an experimental and observational study.
- **Students will learn the basic vocabulary of experimental design.** This will include descriptive statistics, inferential statistics, sample, population, convenience samples, opportunistic samples, non-response surveys, simple random samples, undercoverage, combinations, unbiased estimators and the Central Limit Theorem. *Financial Algebra 2E* section 9-1 includes these topics, normally found in the AP Statistics curriculum. Project 1.3 will have students surveying students in their school to empirically test a famous mathematical finding that defies intuition.
- **Students will determine which measures of dispersion are unbiased estimators.** Project 1.1 will have students determine via sampling with hypothetical populations, which measures of central tendency are unbiased estimators.

- **Students will determine which measures of central tendency are unbiased estimators.** Project 3.5 will have students determine via sampling with hypothetical populations, which measures of central tendency are unbiased estimators.
- **Students will use a random number table to select subjects for a study.** *Financial Algebra 2E* section 9-1 shows students how and why random numbers tables are used. A table of random numbers appears in the Appendix of the book. Project 3.3 requires students to create their own random number table experimentally.
- **Students learn the logic of hypothesis testing.** In *Financial Algebra 2E* section 9-2, students will employ probability and the normal curve to judge results as being probable or improbable. This is the cornerstone of hypothesis testing.

Unit 4: Employment and Income Taxes

In this unit, students will learn about looking for employment, salary, paychecks, deductions, benefits, and Social Security payments. Verbal situations transformed into algebraic expressions and spreadsheets are used to model hourly pay and overtime pay, and piecewise functions are used to model commissions and royalties, and piecework pay. Students learn using the FICA tax function how the graph of a piecewise function can have segments with two different slopes meet at a cusp. Discontinuities are scrutinized as they relate to unfair tax formulas.

Students also explore, model, graph and interpret the Internal Revenue Service's tax tables, schedules, and worksheets by using linear and polygonal functions that have cusps and different slopes over different domains. They will express those domains using compound inequality notation and interval notation. In addition, the topics of function continuity and jump discontinuity will be explored as they relate to the functions created to model the tax bracket calculations. Students will learn how these piecewise functions and polygonal graphs can be used when filing the three major IRS tax forms.

Common Core State Standards for Mathematical Content that are Addressed: A-CED1, A-CED2, A-CED3, A-CED4, A-REI3, A-SSE1, F-BF1, F-BF2, F-IF1, F-IF2, F-IF4, F-IF7b, F-IF8, F-LE1, A-SSE1
CCSS Mathematical Practice Standards addressed: MP1, MP4, MP5, MP6, and MP7

Mathematics Learning Goals Cross-Referenced with Textbook and Projects

- **Students will determine and interpret domains of piecewise functions**. *Financial Algebra 2E* sections 6-1, 6-2, 4-1, 5-3, 10-3 and 11-2 require students to model real-life situations using piecewise functions. Project 3.2 involves modeling the FICA tax function, and interpreting its cusp and its slopes over different domains.
- **Students will graph piecewise functions with different slopes that create cusps**. *Financial Algebra 2E* section 5-5 and Project 3.2 require students to interpret the slope as a rate and use the units of the rate to explain what the graph shows.
- **Students will compute measures of central tendency and rational functions that model average value**. *Financial Algebra 2E* section 5-4 analyzes pension computations

by expressing the mean as a rational function used to compute average salaries that determine pension amounts.

- **Students will use geometric sequences and identify common ratio r**. *Financial Algebra 2E* section 5-2 features the "penny doubled every day" problem to show the power of exponents. A pay scale that defies mathematical intuition is designed to show exponential growth with common ratio 2. Students will find the n^{th} term of a geometric sequence.
- **Students will use the point-slope form of the equation of a line and convert to slope-intercept form**. *Financial Algebra 2E* section 6-1 models the tax worksheet and derives the instructions on the tax worksheet using the piecewise function they create to model the tax schedules. Project 4.1 highlights the conversion of one form to the other and shows how slope intercept form is used by the IRS on the tax worksheet. The ease of computation inherent in the slope-intercept form sacrifices information portrayed in the point-slope form about income thresholds and tax brackets.
- **Students will graph continuous polygonal functions with multiple slopes and cusps**. *Financial Algebra 2E* sections 6-1 and 6-2 uses continuous polygonal functions to model tax schedules. The cusp represents the points at which the tax brackets change. Project 4.2 features continuous functions with cusps derived from the FICA tax function. Students interpret the horizontal component, and its slope as a rate.
- **Students will translate verbal expressions into literal rational, exponential, and linear equations**. *Financial Algebra 2E* sections 5-1 through 5-5 and 6-1 through 6-5, and in fact, all sections of the text, require students to frequently use equations to represent verbal situations. This is a necessary cornerstone of any math modeling course.
- **Students will express domains using compound inequality notation and interval notation**. *Financial Algebra 2E* sections 6-1 and 6-2 use compound inequalities and interval notation to model the brackets on the income tax schedule. The tax brackets are defined by inequalities and students need to differentiate between "less than" and "less than or equal to" by reading carefully.
- **Students will express domains using tax schedule notation**. *Financial Algebra 2E* section 6-2 has students analyzing the IRS verbiage and compound inequalities, translating from one to the other.
- **Students will interpret jump discontinuities**. *Financial Algebra 2E* sections 5-5 and 6-2 use continuous functions with cusps to model the tax schedule. Students analyze this to explain the unfairness of a jump discontinuity when it is interpreted in the context of moving into another tax bracket.
- **Students will model a tax bracket, given a compound inequality statement, and model a tax bracket to determine the tax using a linear equation**. *Financial Algebra 2E* section 6-2 has the students computing taxes form the piecewise function, and from the linear function in slope-intercept form used on the IRS tax worksheet.
- **Students will write equations in point-slope form.** *Financial Algebra 2E* section 6-2 shows how the slope is both the tax rate and the coefficient of the parenthetical expression that shows at what income that tax rate is effective.
- **Students will create and interpret piecewise functions and give the domains and literal interpretations of the algebraic model**. *Financial Algebra 2E* section 6-2 uses tax schedules to investigate piecewise functions. Project 3.1 allows students to see how the IRS made it easier for taxpayers by translating the piecewise function into a worksheet deliberately driven by words, not algebra.

- **Students will graph piecewise functions and determine the cusps of piecewise functions from the function notation**. *Financial Algebra 2E* section 6-2 shows how the tax bracket rates become slopes when equations are created and graphed. Where slopes change, the coordinates of the cusp show the income levels at which tax rates change.
- **Students will adapt all algebraic formulas in the unit for use in spreadsheets.** *Financial Algebra 2E* sections 6-2 and 5-3 require students to adapt the structure of algebraic thinking to the language understood by the spreadsheet program. This requires intense attention to precision since any deviation from the correct syntax will prevent the spreadsheet from computing correctly. The spreadsheets will be computing payroll and computing taxes.

Unit 5: Automobile Ownership

Various functions, their graphs, and data analysis can be instrumental in the responsible purchase and operation of an automobile. In this unit, students will model auto sales and purchases using logarithms, frequency distributions, modified box and whisker plots, stem and leaf plots, and linear and curvilinear regression. Deprecation is modeled through arithmetic and geometric sequences, and commission and classified ad rates are modeled through piecewise functions. The probability inherent in auto insurance is modeled using conditional probability, two-way tables, independent events and Venn diagrams. Projectile motion, irrational functions and parabolas are used to model accident deconstruction.

Common Core State Standards for Mathematical Content that are Addressed: A-CED2, A-CED3, A-CED4, A-REI2, A-REI11, A-SSE1b, A-SSE3, F-IF1, F-IF2, F-IF3, F-IF4, F-IF6, F-IF7a, F-IF7b, F-IF7e, F-IF8b, F-IF9, F-BF2, F-BF5, F-LE1b, F-LE1c, F-LE2, F-LE4, F-LE5, G-C5, S-ID1, S-ID2, S-ID3, S-ID4, S-ID6, S-ID7, S-CP1, S-CP2, S-CP-3, S-CP4, S-MD1, S-MD2, S-MD4
CCSS Mathematical Practices addressed: MP1, MP2, MP3, MP4, MP5, MP6.

Mathematical Learning Goals Cross-Referenced with Textbook and Projects

- **Students will transform raw data into a frequency distribution**. *Financial Algebra 2E* section 4-2 shows students how to make sense out of large collections of data relating to automobile sales and purchases using frequency distributions. Students will compute measures of central tendency including the mean, median and mode, and explain appropriate uses of each. Project 5.1 asks students to chart internet automobile data and discuss the frequency distribute of that data.
- **Students will model situations using arithmetic sequences.** *Financial Algebra 2E* section 4-5 offers students an automobile linear depreciation situation in which the car values can be modeled using an arithmetic sequence. Students are given values for two different years and are asked to use the general equation of an arithmetic sequence to determine the common difference. This common difference will be the yearly depreciation amount. Project 5.2 asks students to research how the IRS handles the calculation and reporting of depreciation deductions.

- **Students will model situations using geometric sequences.** In *Financial Algebra 2E* section 4-6, students model exponential depreciation over time using geometric sequences. They determine a common ratio and then the general geometric sequence equation that models the car values. Using that equation, they determine a value for a future date. Project 5.5 asks students to do research on the historical values of cars to determine the depreciation percentage, set up a geometric sequence, and use the general form of the geometric sequence to identify auto values for given ages.
- **Students will create and interpret stem and leaf plots and side-by-side steam plots that display two distributions simultaneously.** *Financial Algebra 2E* section 4-2 illustrates ways to chart data related to car sales, purchases and insurance. Stem and Leaf plots are offered as an alternative to frequency distribution lists. Students will compute measures of central tendency including the mean, median and mode, and explain appropriate uses of each. Project 5.1 has students construct stem and leaf plots to analyze automobile prices.
- **Students will create and interpret side-by-side, modified box and whisker plots, and learn how to display them on a graphing device.** *Financial Algebra 2E* section 4-2 offers students contextual opportunities to compute measures of dispersion including the range, the quartiles, the interquartile range and the boundaries for outliers using the expressions $Q_1 - 1.5(IQR)$ and $Q_3 + 1.5(IQR)$. Project 5.1 has students construct box and whisker plots to analyze automobile prices.
- **Students will use linear and exponential regression analysis to model automobile related situations.** *Financial Algebra 2E* section 4-6 offers students opportunities to investigate automobile depreciation using regression analysis. Project 5.1 has pair mileage and price data of similar make and model cars to create scatterplots and identify the regression line in order to make a presentation to the class about the predictive value of regression analysis as it pertains to used car purchases.
- **Students will create and interpret piecewise (split) functions.** *Financial Algebra 2E* section 4-1 introduces students to continuous piecewise functions based on classified ad costs and commission payment schedules. Domains and cusps are identified and interpreted. Students will graph piecewise functions using mutually exclusive domains.
- **Students will use conditional probability to interpret two-way tables.** *Financial Algebra 2E* section 4-4 covers the conditional probability formula, and how probabilities can be computed from two-way tables.
- **Students will be able to determine whether or not two events are independent.** The concept of conditional probability is used to define independent events in *Financial Algebra 2E* section 4-4.
- **Students will create and interpret Venn diagrams.** The relationship between frequencies, Venn diagrams, and conditional probability is discussed in *Financial Algebra 2E* section 4-4.
- **Students will model automobile depreciation using linear function models.** *Financial Algebra 2E* section 4-5 employs the Internal Revenue Service linear depreciation method to model car depreciation over time. Students contextualize slope, intercepts and function values. Project 5.2 asks students to use linear and exponential graphing techniques in the context of automobile depreciation.
- **Students will create, use, and interpret linear systems of equations.** *Financial Algebra 2E* section 4-5 addresses the interrelatedness of the automobile linear expense and linear depreciation functions. Students contextualize system domains and the

intersection point. Project 5.2 asks students to use linear and exponential graphing techniques in the context of automobile depreciation.

- **Students will model exponential depreciation as $y = Px^b$ where P is the purchase price and $x < 1$, and compare the depreciation to an increasing linear expense function.** *Financial Algebra 2E* section 4-6 develops the algebraic representation of a depreciation percentage and uses it in the exponential depreciation equation. Students set up linear/exponential systems and contextualize the domains and point of intersection. Project 5.2 asks students to use linear and exponential graphing techniques in the context of automobile depreciation.
- **Students use logarithms to determine the age of a car given the depreciation ratio, the car value, and original price.** In *Financial Algebra 2E* section 4-6, students are presented with situations in which they are given the exponential depreciation equation for a car and asked to determine the age given specific data. In order to solve for the exponent, students must use the laws of logarithms.
- **Students will set up and use algebraic ratios.** *Financial Algebra 2E* section 4-7 requires students to analyze driving data by setting up rational functions.
- **Students will create and use the total stopping distance function which is created by adding a linear reaction distance function to a quadratic braking distance function.** *Financial Algebra 2E* section 4-8 models driving safety data by setting up and using linear and quadratic functions as components of the stopping distance formula. Project 4.3 examines the physics of driving as it relates to the stopping distance function.
- **Students will use and interpret a square root function.** *Financial Algebra 2E* section 4-9 offers students the skid speed square root function. Data from accidents are analyzed using this function. Project 5.4 examines the physics of driving as it relates to the square root skid speed function.
- **Students will use geometry theorems involving chords intersecting in a circle and radii perpendicular to chords to determine yaw mark arc length.** *Financial Algebra 2E* section 4-9 expands the use of the square root function to introduce the formula for determine the speed of a vehicle based on yaw marks. The yaw mark skid speed function is a composite function $f(g(x))$ where $f(x)$ is a square root function and $g(x)$ is a quadratic function. Students find the radius of the yaw and compute the arc lengths. Project 5.4 examines the physics of driving as it relates to the yaw speed function.
- **Students will adapt all algebraic formulas from the unit for use in spreadsheets**. *Financial Algebra 2E* sections 4-7 and 4-9 ask students to create automobile linear and quadratic, exponential and square root formulas for use in spreadsheets.
- **Students will examine projectile motion problems as they relate to automobile safety.** In *Financial Algebra 2E* section 4-9, students are presented with situations in which they will use Newton's first law of motion and quadratic equations to analyze projectile motion problems.
- **Students will compute outliers and which measures of central tendency are resistant to outliers.** In *Financial Algebra 2E* section 4-2 students will construct the 'fences" for the outliers, employ them in modified boxplots, and determine who outliers affect mean, median, and mode.

Unit 6: Consumer Credit

Credit comes at a price and in this unit, students learn how to use mathematics to make wise credit choices that fit their needs, current financial situation, and future goals. Students explore loan information and model that data using regression analysis to find the linear, quadratic, cubic, and exponential equation of best fit. They use exponential and rational functions in the forms of the simple interest formula, the monthly payment formula, and the simplified interest formula to determine total costs of borrowing for an education and large purchases. Students will use logarithms and logarithmic operations in order to determine loan lengths that meet individual consumer needs.

Common Core State Standards for Mathematical Content that are Addressed: A-CED1, A-CED2, A-CED3, A-CED4 , A-REI2, A-REI3, A-REI4b, A-REI6, A-REI7, A-REI10, A, REI11,A-REI12, A-SSE1, F-IE4 , F-IF1, F-IF4, F-IF5, F-IF7a, F-IF8, S-ID6 , N-Q1, N-Q2, N-Q3, N-CN , S-ID8, S-ID9, S-IC1, S-IC3, S-IC5
Common Core Standards for Mathematical Practice addressed: MP1, MP2, MP3, MP4, and MP5.

Mathematics Learning Goals Cross-Referenced with Textbook and Projects

- **Students will create, evaluate, interpret and solve algebraic proportions.** *Financial Algebra 2E* sections 3-1 and 3-2 utilize ratios and proportions in the context of short term and long-term credit situations.
- **Students will model situations using linear, quadratic, cubic, and exponential equations.** *Financial Algebra 2E* sections 3-2 and 3-4 ask students to explore loan information and model that data using regression analysis to find the line or curve of best fit.
- **Students will create, use, and interpret exponential growth and decay equations that model given situations.** *Financial Algebra 2E* sections 3-2, 3-3 and 3-4 examine exponential equations in the context of loans. Students apply an exponential equation in the form of the monthly payment formula where the exponent is present in both the numerator and the denominator. Project 6.1 has students using exponential functions to analyze a consumer credit situation.
- **Students will use the slope-intercept form y=Mx+b where M is the exponential monthly payment equation.** *Financial Algebra 2E* sections 3-2, 3-3 and 3-4 offer students opportunities to explore the monthly payment function. Project 6.1 has students use the monthly payment formula as a means to analyze a consumer credit situation.
- **Students will use exponential and rational functions to learn about student loans.** In *Financial Algebra 2E* Section *3-3* student examine the different types of student loans available and the financial obligations and ramifications of each type. They use the simple interest formula, the monthly payment formula, and the simplified interest formula to determine total costs of borrowing for an education. In Project 6.3, students investigate the variety of student loan options available to them as they apply the mathematics used in the section.

- **Students will use model and calculate the finance charge using the exponential monthly payment formula and the retail price.** *Financial Algebra 2E* sections 3-4 through 3-7 examine loan and credit card transactions and statements using the monthly payment formula. Project 6.2 asks students to calculate a finance charge by using exponential functions as they deconstruct a credit card statement.
- **Students will use inverse functions to create the logarithm and natural logarithm function.** *Financial Algebra 2E* section 3-4 applies logarithms as a means of solving for the time, t, in the monthly payment formula to determine the length of a loan. Project 6.1 has students using logarithmic functions to analyze a consumer credit situation.
- **Students will interpret and use summation notation to model the average daily balance.** *Financial Algebra 2E* section 3-7 uses summation to deconstruct the entries on a monthly credit card statement. Students will calculate the average daily balance and finance charge using the summation notation formula. Project 6.2 asks students to use summation in order to determine the finance charge.
- **Students will create and use algebraic formulas and apply them for use in spreadsheets.** *Financial Algebra 2E* sections 3-1, 3-2, and 3-6 ask students to create linear, exponential and logarithmic formulas in the context of consumer credit for use in spreadsheets. Project 6.2 asks students to create a spreadsheet that models a credit card statement situation.

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Unit 7: Independent Living

In this unit, students work their way through the mathematics that models moving, renting, and purchasing a place to live. They also explore the geometric demands of floor plans, areas of shaded and irregular figures, apothem, and discover the relationship between area and probability. Trigonometric functions, the Pythagorean theorem, slope, and similar triangles are used to model ladder safety, deck building, and proximity to falling trees. Rational functions with multiple independent variables are used to model air conditioning BTU requirements.

Common Core State Standards for Mathematical Content that are Addressed: A-APR6, A-CED2, A-CED3, A-LE1, A-REI6, A-SSE1, F-BF1, G-C5, G-MG3, S-ID6a, S-ID6c, S-ID8, G-SRT2, G-SRT6, G-SRT7, G-SRT8
Common Core Standards for Mathematical Practice addressed: MP1, MP4, MP5.

Mathematics Learning Goals Cross-Referenced with Textbook and Projects

- **Students will use systems of linear equations to model a given situation.** *Financial Algebra 2E* section 7-1 asks students model a home move situation using a system of linear equations.

- **Students will use linear regression to model a given situation and make predictions using the regression equation.** *Financial Algebra 2E* section 7-1 offers students an opportunity to determine a regression equation that best fits data representing the square footage and monthly rent of similar apartments.
- **Students will use rational functions with multiple independent variables**. *Financial Algebra 2E* section 7-3 asks students to set up and interpret rational functions representing back-end, front-end, and debt-to-income ratios for mortgage applications. In sections 8-3 and 8-4 students use the rational function representing the monthly loan payment to investigate home loan costs. Project 7.4 asks students to set up rational functions in order to determine the reduction in interest from extra mortgage payments.
- **Students will use a rational function containing exponents.** *Financial Algebra 2E* section 7-5 offers students opportunities to use the monthly payment rational function in the context of mortgage points. Project 7.5 asks students to research points purchases at three different lending institutions. Then, using the monthly payment rational function, determine at which institution is makes the best financial sense to purchase points.
- **Students will use the apothem to derive the formula for the area of a regular polygon, and apply the formula.** *Financial Algebra 2E* section 7-2 focuses on floor plans. Students learn how to use the apothem to calculate the square footage of a regular polygonal plot. Project 7.2 has students work with floor plan blueprints to determine areas of plane figures that have sections removed from them. Project 6.3 offers students an opportunity to determine areas by first finding the apothem.
- **Students will use probability, proportions and the Monte Carlo Method.** *Financial Algebra 2E* section 8-2 discusses the use of the Monte Carlo method to determine the area of an irregular shaped plot of land. Project 7.1 uses the Monte Carlo method to find the area of an irregular landscape design.
- **Students will use dilation factors**. *Financial Algebra 2E* section 7-2 has students drawing and interpreting house plans to scale. Students will solve scale drawing problems using proportions.
- **Students will compute areas of irregular and shaded regions**. *Financial Algebra 2E* section 7-2 reviews area formulas and area processes for regular and irregular regions.
- **Students will use multi-variable rational functions.** *Financial Algebra 2E* section 7-2 uses multi-variable rational functions to compute air conditioner BTU needs.
- **Students will create and use exponential equations.** *Financial Algebra 2E* section 7-6 uses exponential equations and exponential regression equations to model percentage rent increases.
- **Students will adapt all algebraic formulas for use in spreadsheets.** *Financial Algebra 2E* sections 7-3, 7-4, and 7-6 offer students opportunities to model home ownership situations using spreadsheets.
- **Students will use the Pythagorean Theorem.** Homeowner applications of the Pythagorean Theorem are covered in *Financial Algebra 2E* section 7-7. Students also use the converse of the Pythagorean Theorem to find right angles.
- **Students will use similar triangles and trigonometry to solve homeowner problems.** *Financial Algebra 2E* section 7-7 employs trigonometry in solving landscaping and tree trimming problems using right triangle trigonometry.
- **Students will use slope to examine roof pitch.** Students will define the pitch of a roof as rise over run and use the slope formula in *Financial Algebra 2E* section 7-7.

Unit 8: Retirement Planning and Budgeting

The focus of this unit is on the mathematics of fiscal plans that workers can make years ahead of their retirement date, including a detailed study of retirement savings plans, both personal and Federal, employee pension programs, and life insurance. Rational functions are used to model present value, future value, and periodic investments. The intricate formula for computing Social Security benefits is scrutinized, and investment diversification is modeled using graphs. Probability and expected value are used to compute how life insurance companies can earn a profit.

Students will also explore how rational functions can be used to model average costs over time of major purchases and services as they create, graph and interpret the functions both singly and as a system. They will also employ the greatest integer function as part of a piecewise function that is used to model household expenses over time. The unit culminates with the creation of a budget, modeling budget situations with a budget-line system, and organizing/interpreting budget information through the use of matrices and matrix operations.

Common Core State Standards for Mathematical Content that are addressed: A-CED3, A-REI10, A-SSE1, F-BF1, F-IF4, F-IF5, F-IF7a, F-IF7b, F-IF7d, F-IF8b, N-Q1, N-Q2, N-VM6, N-VM7, N-VM8, N-VM-9, N-VM10, S-MD1, S-MD2, S-MD4, S-MD5
Common Core Standards for Mathematical Practice addressed: MP1, MP2, MP4, MP5, MP6, and MP8

Mathematics Learning Goals Cross-Referenced with Textbook and Projects

- **Students will use the exponential future value of a periodic investment formula to predict balances after t years when given a periodic deposit amount, an investment return rate, and compounding information**. *Financial Algebra 2E* sections 2-7 and 10-1 teach students how to compute the future value of an account in which the same periodic investment is made over a long period of time.
- **Students will use the exponential present value of a periodic investment formula of the form to determine the principal when given a future value, a time in years, an investment return rate, and compounding information**. *Financial Algebra 2E* sections 2-8 and 10-1 show students how much they have to contribute over a certain length of time to reach a future financial goal. This is part of Project 8.2, in which students act as financial planners.
- **Students will use rational functions to plan retirement funding**. *Financial Algebra 2E* section 10-1 uses multivariable rational functions to compute requirements to meet specific financial goals. This is also a component of Project 8.2.
- **Students will use inequalities to define domains when creating algebraic expressions**. *Financial Algebra 2E* section 10-1 expresses income brackets as compound inequalities which serve as the domain for tax functions.
- **Students will create and analyze discrete probability distributions.** *Financial Algebra 2E* section 10-4 requires students to set up a table representing a discrete probability distribution by filling in the values of a random variable and the associated probabilities.

Project 8.1 uses the mortality tables to compute life insurance benefits at different periods in the policy of a term insurance policy.

- **Students will write rational expressions to represent increase over time.** *Financial Algebra 2E* section 10-4 requires students to express percent increases in life insurance premiums as rational functions.
- **Students will use and interpret the greatest integer function.** *Financial Algebra 2E* section 11-2 introduces the greatest integer function to algebraically model cell phone charges.
- **Students will determine and interpret the expected value of a probability distribution.** *Financial Algebra 2E* section 10-4 develops the concept of expected value, in the context of life insurance. The random variable is the life insurance benefit. Mortality tables supply the probabilities associated with each value of the random variable.
- **Students will create interpret, and graph greatest integer functions**. *Financial Algebra 2E* section 11-2 incorporates the greatest integer function to model billing that is dependent on entries being integers.
- **Students will incorporate the greatest integer function into a piecewise function.** *Financial Algebra 2E* section 11-2 combines piecewise functions with the greatest integer function when the criteria involved in the splitting of the function depends on whether the input variable is an integer or not.
- **Students will create, interpret, and graph a system of a linear and a piecewise function and determine the point of intersection.** *Financial Algebra 2E* section 11-2 compares two optional billing plans offered by a utility to determine at what usage the costs are equivalent. This would be one facet of the many variable inherent in the completion of Project 8.4.
- **Students will use sectors and central angles of a circle to depict proportional categories on a pie chart when given categorical information**. *Financial Algebra 2E* section 11-3 requires the students to proportionally divide a circle into sectors that represent given percentages, to display budget allocation.
- **Students will create and interpret linear budget line equations and inequalities to optimize purchases within a given monetary constraint**. *Financial Algebra 2E* section 11-3 features these graphs. Project 8.4 investigates the combinations of possible purchases within a constrained budget.
- **Students will use multiple representations to chart data relating to retirement and budgeting**. *Financial Algebra 2E* sections 11-3 and 11-4 uses a matrix to chart budgeting for a household over a year. Project 8.3 uses spreadsheets to calculate cash flow, net worth, and debt reduction.
- **Students will model budget situations using matrices, matrix addition, scalar multiplication, and matrix multiplication.** In *Financial Algebra 2E* section 11-5, students will analyze budgetary situations and circumstances by using matrix operations.

(C)
ACTIVITIES, MAJOR RESOURCES, MATERIALS AND INSTRUCTIONAL STRATEGIES TO BE USED

SAMPLE ACTIVITIES

The activities presented in this section are all verbal problem-solving activities that relate to the unit being studied. Students must represent the verbal situation symbolically, manipulate those symbols to arrive at an answer, and then interpret that answer in the context of the problem. This offers students opportunities to make sense of quantities and their relationships within those problem-solving settings through multiple representations. Students can approach, access, and deconstruct the necessary mathematics using handheld graphing utilities, manipulatives, spreadsheets, and/or graphing websites such as DESMOS. The assignments throughout this course require students to attend to precision in their responses both in the computational and algebraic fluency required to arrive at those answers and in the unit-material used to contextualize the answers.

The prevalence of mathematical modeling assignments allows students to practice seeking out mathematical structure in what may seem to them to be an unstructured situation. Identifying and exploiting the structure leads students to a richer understanding of the themes and regularities that are present in the real world. Students make tables, find patterns, and offer conjectures based on the patterns. This form of inductive reasoning is a cornerstone of mathematical thinking. The assignments and other course-related activities optimize students' exposure to extrapolating what they have learned to routine and non-routine mathematically-dependent situations they encounter in their futures.

Most assignments require the student to prepare a presentation on their finished work. This can be a PowerPoint show, a webinar, a poster presentation, or a presentation using transparencies. The student audience gets to critique the presentation, ask questions, and make comments, in a firmly established, constructive, positive "safe" zone. The presentation is graded, and the quality of student critiques and comments can also be graded.

There are three types of sample activities offered for each of the units. They are as follows:

- **Projects –** Projects are an engaging form of alternative assessment. They give students additional avenues to show what they have learned. They can be library-research based, internet-research based, and/or community-research based. Projects can be presented to the class and/or displayed for the class, school, or community to see.

- **What's The Problem? -** Here, students are offered a modeling opportunity to develop and strengthen problem posing skills. They are given a context and a completed equation used to solve a problem relating to that context. They are then asked to develop the question (problem) that relates to the solution equation. Once students have completed the activity, have them share their results with the class looking for commonalities and differences in what others determined to be the associated problem.

- **You Write The Story? -** Students are given a non-traditional graph related to the topic they have just been studying. They are asked to write a short, news-type article centered on the data depicted in the graph.

Unit 1: Discretionary Expenses

PROJECTS

Project 1.1: Using Statistics to Analyze Auto Prices

Mathematics: Measures of central tendency, distribution charts
Mathematics Learning Goals: To use measures of central tendency and distribution charts to analyze automobile prices.

Students choose a make, model and year for an automobile. They use the Internet and newspaper classified ads to find 20-30 of those cars for sale. They get the price of the car and the mileage it has. They then determine the mean, median, and mode car prices and mileage amounts. Students should identify 10 price intervals which encompass all of the car prices and 10 mileage intervals which encompass all of the odometer readings at time of sale. Using these intervals, students create charts depicting the frequency, relative frequency, cumulative frequency, and relative cumulative frequency for the prices and mileages. Their results are presented to the class via PowerPoint presentation or poster presentation.

Project 1.2: Can a Basketball Team Have Normally Distributed Heights?

Mathematics: Normal distribution, measures of central tendency and dispersion.
Mathematics Learning Goals: To understand shifts of the normal distribution.

Are the heights of National Basketball Association (NBA) players are normally distributed? Students might intuitively think not since all basketball players are tall. They will need to get the heights of all NBA players by using the Internet. They will research what a normal probability plot is and see how their calculators can create one. Then they will create a normal probability plot and interpret it.

Project 1.3: The Famous Birthday Problem

Mathematics: Surveying, relative frequencies, probabilities.
Mathematics Learning Goals: To determine why and how the answer to this problem defies mathematical intuition.

A very famous problem in statistics is the "Birthday Problem." Students will be asked to answer the problem before embarking on an empirical quest to find the answer. Students will poll classes in school and compile data on birthdates. They then need to determine what percent of the

classes had matching birthdates. After the experimental approximation of the solution, students will then research, interpret, and explain the theoretical solution. They will them explain why the problem is so mathematically deceptive.

Project 1.4: Using Relative Frequency to Make Predictions

Mathematics: Relative frequency, empirical probability.
Mathematics Learning Goals: To determine how random selections can lead to a prediction on the frequency of items in a population.

Students will randomly select, with replacement, colored candy items, one at a time, from a cup. They repeat this one hundred times and record their findings. They will set up a relative frequency table for each color, and us e this information to predict, given the total number in the population of candies on the cup, the frequency of each colored candy in the cup.

Project 1.5: Using Regression to Predict Spending

Mathematics: Bivariate data, scatterplots, regression curve of best fit, correlation.
Mathematics Learning Goals: To use regression analysis to interpret trends and make predictions about spending and expense data.

Use any statistics website to make a list of spending data by age of the consumer, income of the consumer, year, or any other numerical factor. List at least 10 ordered pairs of data. Make a scatterplot of the data and identify any trends that you see. Then do a linear, quadratic, and cubic regression analysis to determine the curves of best fit in each situation. Finally, pick a dependent variable amount outside of the range you selected and use the regression equations to make a spending prediction based on that amount.

Project 1.6: Cellphone Use While Driving Data Analysis

Mathematics: Data collection, analysis, interpretation, display and reporting
Mathematics Learning Goals: To use sample data analysis to interpret a situation and compare that analysis to research findings.

It has been well documented that driving while on a cell phone is distracting, whether or not it is legal in your state. For this project you will need a team of eight or more participants. You are going to pick a busy intersection in your neighborhood. Stand on the sidewalks only! You need one person for each direction cars are moving, to count the number of cars that pass in an hour. You will need a separate person for each direction to count the number of drivers using cell phones as they drive. Spend an hour or more compiling data and create a PowerPoint summarizing all of your statistical findings. Find some Internet research on studies about the dangers of distracted driving and include those findings in your research. Decide as a group if distracted driving is an issue in your community.

Project 1.8: Dispersion Analysis of Airline Fares

Mathematics: Data collection, analysis, interpretation, display and reporting
Mathematics Learning Goals: To use sample data analysis to interpret a dispersion of data situation
How dispersed are airline prices? Pick a departure and a destination city. Airline prices fluctuate tremendously, depending on the carrier, the trip length, demand for the trip, the time of year, and how far in advance the flight is. Check Internet prices for the same flights by the same carriers each day for 3 weeks. Compile all of your data and report your findings using frequency tables and measures of spread as part of a PowerPoint presentation.

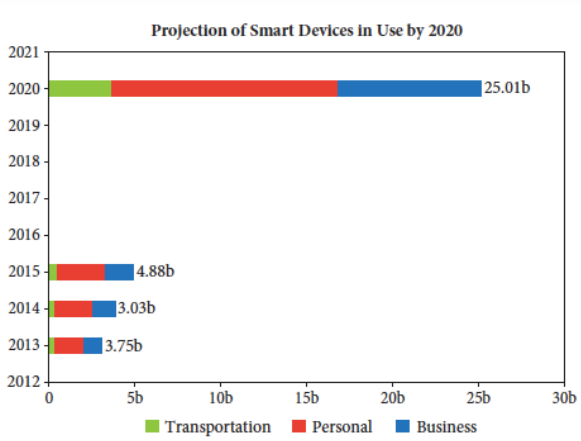
What's The Problem?

Examine the equation below used to determine the mean. Look through your textbook and your notes to help you write a problem that could be modeled by the equation.

$$\frac{4(24.53) + 10(25.00) + 8(27.75) + 3(28.10) + 2(30.00)}{27} = 26.46$$

You Write The Story

Even after home use of the Internet was first introduced in the 1990s, there weren't a lot of people who had access to the World Wide Web. For many, that access was considered a luxury and most definitely a discretionary expense. As years go by and the Internet becomes a bigger part of our lives, there is a real question as to whether it is no longer discretionary but essential. The graph depicts the estimated number of smart connected devices (Internet-connected devices) in use worldwide by category. Write a short, news-type article centered on the data in this graph.



Unit 2: Banking

PROJECTS

Project 2.1: How Interest Method Affects Monetary Growth

Mathematics: Simple interest, compound interest
Mathematics Learning Goals: To determine how increased compounding affects growth.

Students are first introduced to the meaning of compounding numerically via mathematical iteration. Before embarking on a rigorous study of limits and compound interest algebraic formulas, students are asked "How much would $1,000 grow to, in one year, at 100% interest compounded continuously?" The 100% interest and continuous compounding often leads them to guess much higher than the actual amount. Their guesses are recorded, and a statistical analysis of their guesses is made. Outliers are carefully noted. The findings of this activity are scrutinized after students complete Project 2.3.

Project 2.2: Deriving the Compound Interest Formula

Mathematics: Inductive reasoning, exponential functions, rational functions
Mathematics Learning Goals: To use patterns and induction to generate for selected forms of compounding and adapt them to monthly, weekly, daily, and hourly compounding.

Students will compute interest for each interest period over a semi-annual and quarterly compounded account for a given balance and interest rate. They will derive the general algebraic formulas for these two types of compounding. They will then look for patterns in the semi-annual and quarterly compound interest formulas to inductively conjecture about the general formula for compounding. They will then find formulas for monthly, weekly, daily and hourly compounding, and compute and compare the interest earned over one year for these accounts.

Project 2.3: Using Limits to Derive the Natural Base e

Mathematics: Rational functions, exponential functions
Mathematics Learning Goals: To use substitution and patterns to generate a series that approaches e as x approaches infinity.

Students will be introduced to the notion of limits and limit notation and apply it to the compound interest formulas previously derived. They will increase the number of compoundings by first computing interest when the compounding period is every minute, and then every second, for a given balance and interest rate. They will then let the number of compoundings 'n' approach infinity to see what happens to the annual interest as the number of compoundings approaches infinity. They will analyze the compound interest formula without the balance, and explain the "battle" between the base and the exponent of the expression $\left(1+\frac{1}{n}\right)^n$ as $n \to \infty$.

Project 2.4: Future Value and College Costs

Mathematics: Rational functions, regression
Mathematics Learning Goals: To estimate the cost of a college education in 18 years and determine how much needs to be saved each month to have the costs covered by the 18th year.

Students pick a college and find out the cost of tuition, room and board (if necessary) and fees over the past ten years. They set up a regression line or curve of best fit. They then predict the cost of a college education in 18 years (as if they just had a child and were trying to save for college). They then use the prevailing interest rate and the future value formula to determine the monthly periodic deposit that would be necessary to have the full college cost saved by the child's 18th birthday. They then do the problem with interest rates slightly higher than the prevailing rate.

Project 2.5: Finding The Term of an Account Using Logs

Mathematics: Logarithms, common logarithms, natural logarithms
Mathematics Learning Goals: To determine the term of a systematic payment account
Pick three expensive items that you might like to purchase at some time in the future. Determine the total cost of each item with tax and shipping (if applicable). Determine what you think you might be able to afford each month to deposit into a savings account assuming that you would be saving for only one of the three. Find three different savings accounts offered by three different lending institutions. These accounts should compound interest monthly. Determine the APR for each account. Use the formula in section 2-10 for systematic savings to determine how long it would take to save enough money to make the purchases. Write a report on your findings.

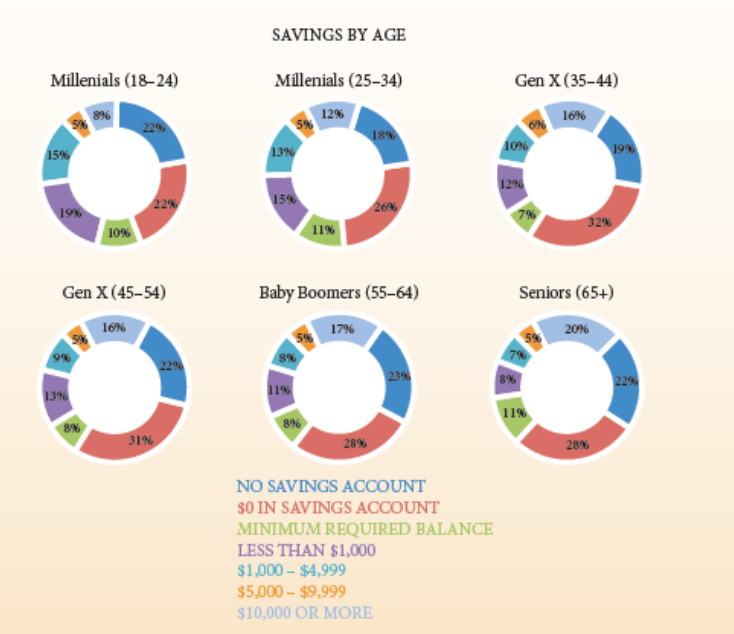
WHAT'S THE PROBLEM?

Examine the equation below used for determining the balance on a bank account. Look through your textbook and your notes to help you write a problem that could be modeled by the equation.

$$2500\left(1+\frac{0.017}{365}\right)^{365(3)}=2630.80$$

YOU WRITE THE STORY!

Examine the color-coded graphs below. It depicts savings account balances by age as of the end of 2015. Write a short news-type article based on the graph. Use the Internet if you need additional information and background to help you explain the graph.



Unit 3: Investing

PROJECTS

Project 3.1: Charting a Corporate Stock

Mathematics: Data Analysis, regression, prediction, modeling, graphical interpretation
Mathematics Learning Goals: The goal of this assignment is to have students use mathematical modeling to chart and interpret stock market trends over a 15-day period. They will make trend predictions based on simple moving average crossover analysis as well as regression models.

Each student selects a corporation traded on the New York Stock Exchange. They produce a background paper, PowerPoint presentation or poster board display on that corporation. Students chart the open, close, high, low and volume data for 15 consecutive trading days. They graph the data using two different formats and then discuss trends that the data shows. They will also calculate three different cluster-lengths of moving averages and, using those clusters, they will create superimposed line graphs. Students discuss trading implications based upon stated domains of graph pairs before and after any intersection points. Finally, they determine the closing price curve of best fit using regression analysis. They must state the regression equation and support why their stated curve best fits the data of closing prices. Students will then use the curve of best fit to predict a closing price on the 16th trading day. They compare that predicted price with the actual closing price on the 16th day and find a percent error.

Project 3.2: Mathematically Modeling A Business

Mathematics: Linear and quadratic functions, linear/linear Systems, linear/quadratic systems, regression analysis
Mathematics Learning Goals: To have students create linear and quadratic models for a start-up business. They will graph and interpret systems of these regression and modeling equations in order to explore the relationship between and among expense, demand, price, revenue and profit.

Students are given a market research scenario for a new product, attained from a focus group questionnaire. The research contains a list of ordered pairs in the form (p,q) where p is a potential price and q is the quantity of the product that the focus group member would purchase if it was set at that price. Using these ordered pairs, students construct a scatterplot, determine the correlation coefficient, and identify a linear regression equation in which q is the independent variable and p is the dependent variable. Then, given information about expenses, they are to set up a linear expense function in terms of the quantity demanded. The quadratic revenue and profit equations are determined and graphed on the same axes with the expense function. Students identify and interpret the breakeven points, the coordinates of the maximum point on the revenue graph, the coordinates of the maximum point on the profit graph, and the price at which the product should be sold in order to maximize profit. Finally, students are told the initial price per share for the company's stock and asked to determine the number of shares that must be sold in order to have enough money to start this business.

Project 3.3: Creating Your Own, Original, Random Number Table

Mathematics: Probability
Mathematics Learning Goals: To create a random number table to help students understand how they simulate random choice.

Students use numbered ping pong balls to create a table of random digits. Aside from using their table to do problems in class, the creation of the table can help them understand that these tables represent random digits and are not "rigged."

Project 3.4: Creating A Financial Portfolio Using Linear Programming

Mathematics: Linear programming
Mathematics Learning Goals: To set up constraint inequalities and a feasible region in order to set up an optimal financial portfolio.

Students should reread Application 8 in section 9-8. This application will be the model for this assignment. They are to assume that they have $500,000 to invest. They should interview a financial advisor and ask for a recommendation of two investment plans. In addition, they should ask what the estimated return on investment for each plan would be. Ask the advisor for constraints on the investment similar to those in Application 8. Then students are to apply the linear programming process to this investing situation and determine the amount to invest in each plan that will yield an optimal return of the investment.

Project 3.5: Unbiased Estimators

Mathematics: Measures of central tendency and measure of spread. Combinations.
Mathematics Learning Goals: To determine which sample statistics are unbiased estimators of population parameters.

Students are given small populations of data. They then compose a table of all possible samples, some with, and some without replacement. They find sample statistics for each sample, and then average the sample statistics. This average is compared to the actual population parameter to see which of the statistics are unbiased estimators.

WHAT'S THE PROBLEM? 1

Examine the equation below used for determining the capital gain made on a stock. Look through your textbook and notes to help you write a problem that could be modeled by this equation.

$((900)(12.55) - (900)(12.55)(0.01)) - ((900)(8.14) + 25) = 3831.05$

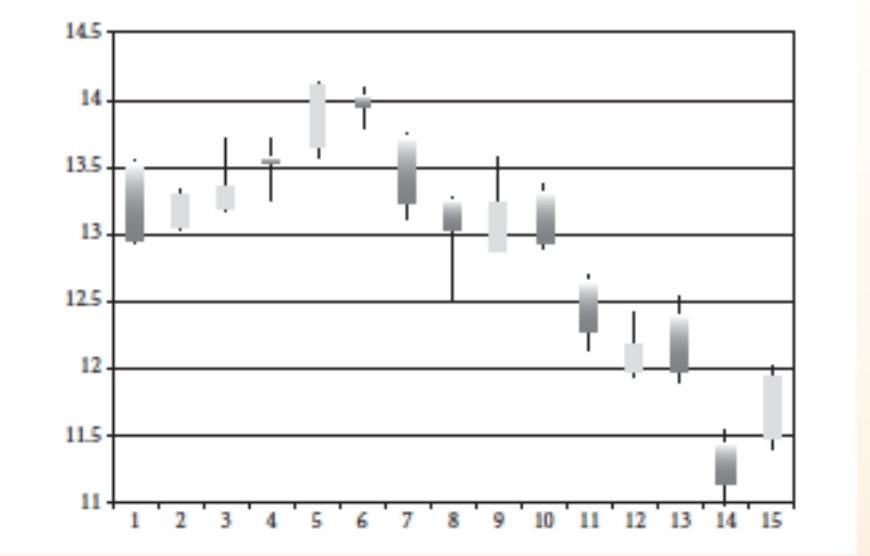
WHAT'S THE PROBLEM? 2

Examine the equation below used for determining the profit on the manufacture and sale of a particular item at a given price. Look through your textbook and your notes to help you write a problem that could be modeled by this equation.

$$(-200(101.50)^2 + 38000(101.50) - (-2600(101.50) + 660000) = 1{,}400{,}450$$

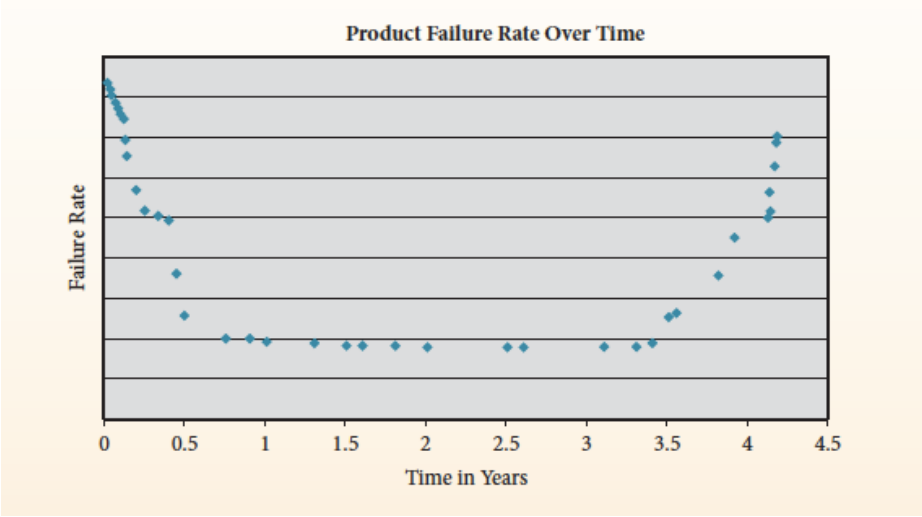
YOU WRITE THE STORY! 1

Examine the graph below. Write a short news-type article using facts obtained online or at the library, centered around this graph.



YOU WRITE THE STORY! 2

A product failure is when a product fails to do what it was manufactured to do at some point during its lifetime. The scatter plot below depicts the rate of failures reported for a product. A scatter plot with this shape is sometimes called a "bathtub curve" because it resembles the cross section of a bathtub. Write a short newspaper-type article centered on the data in this graph, based on a hypothetical situation you create.



Unit 4: Employment and Income Taxes

PROJECTS

Project 4.1: Creating the Tax Worksheet

Mathematics: Domains, piecewise functions, linear functions and graphs, point-slope form, slope-intercept form, graphs with cusps.
Mathematics Learning Goals: To derive the slope-intercept form used on the IRS tax worksheet by translating tax tables into piecewise functions.

The tax tables give taxpayers a function in which the independent variable is the taxable income and the dependent variable is the tax. It is convoluted and has confused taxpayers for years. Within the last decade, the IRS created a worksheet that uses the slope-intercept form of the equations of a line to simplify calculations for the taxpayer. In this Project, students interpret the IRS Schedule, express the domains using compound inequality notation, and create the piecewise function that models the IRS intentions. They then covert this function, which is a translated version of point-slope form, into the slope-intercept form to create the tax worksheet.

Project 4.2: Graphing the FICA Tax Function

Mathematics: Piecewise functions, slope, cusps, linear equations
Mathematics Learning Goals: To use graphs to compare the FICA tax longitudinally over a prescribed number of years.

Students look up the FICA tax percents, and maximum taxable incomes to create piecewise functions for each of the last six years. They compute the maximum FICA tax, and graph all six years on the same axes, and use the graph to write a paragraph on what has happened to FICA taxes over those years. They discuss the significance of the coordinates of the cusp. They do the same for the tax years 1981-86 and compare the last six years to the years 1981-1986. The assignment is replicated using the Medicare tax percent.

WHAT'S THE PROBLEM? 1

Examine the equation below used for payroll deduction calculations. Look through your textbook and your notes to help you write a problem that could be modeled by the equation.

$$4911.30 = 0.062(64200) + 0.0145(64200)$$

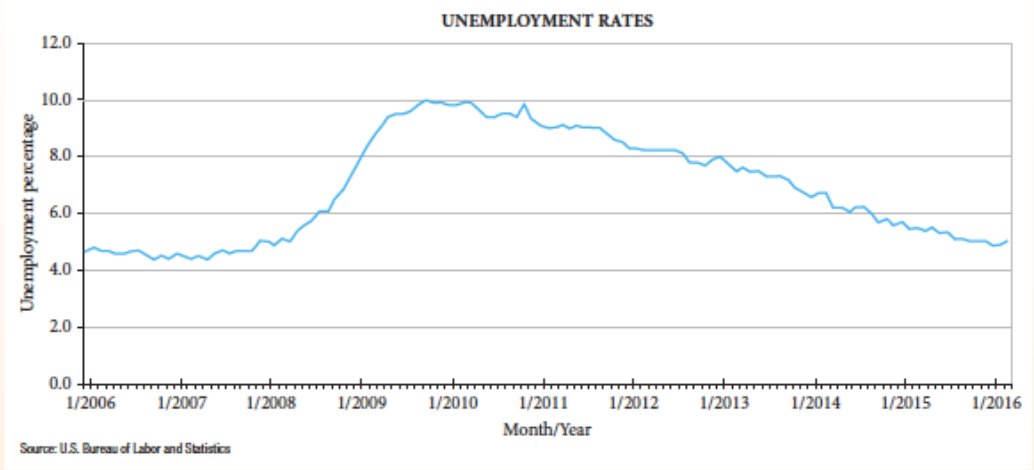
WHAT'S THE PROBLEM? 2

Examine the equation below used for determining the tax owed by a married, filing jointly taxpayer. Look through your textbook and your notes to help you write a problem that could be modeled by this equation.

$$66381.30 = 51577.50 + 0.33(275{,}310 - 230{,}450)$$

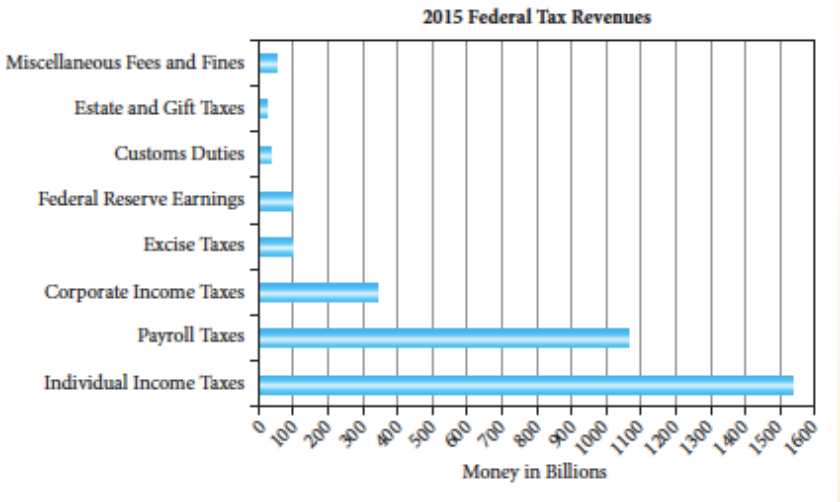
YOU WRITE THE STORY! 1

Write a short news-type article based on the graph below.



YOU WRITE THE STORY! 2

Examine this bar graph. Write a short news-type article based on the graph below.



Unit 5: Automobile Ownership

PROJECTS

Project 5.1: Using Statistics to Negotiate Auto Transactions

Mathematics: Bivariate data, **c**orrelation, regression, mean, median, mode, quartiles, interquartile range, outliers, modified box-and-whisker plots, stem-and-leaf plots, frequency distributions, scatterplots.
Mathematics Learning Goals: To use measures of central tendency and measures of dispersion to mathematically negotiate the buying and/or selling of an automobile.

Students choose a make, model and year for an automobile. They use the Internet and newspaper classified ads to find 10-20 of those cars for sale. They get the price of the car and the mileage it has. They construct modified box-and-whisker plots and describe the frequency distribution. They pair each car's price with its mileage to create a scatterplot. They classify the association as positive or negative. They find the regression line and correlation coefficient and interpret the relationship as strong, moderate or weak, and discuss its linearity. Their results are presented to the class via PowerPoint presentation or poster presentation.

Project 5.2: Automobile Cost and Depreciation

Mathematics: Exponential regression, **g**raphing linear and exponential functions, rational functions, linear/exponential systems, systems of linear equations, slope-intercept form.
Mathematics Learning Goals: To use graphing techniques to compare the value of a car to the expense of purchasing it throughout its lifetime.

Using the monthly payment rational function, students graph the cost C of purchasing a new car, using the down payment as the y-intercept, and the monthly payment as the slope. They then investigate three types of depreciation: straight-line, exponential, and historical bath tub graphs. They graph the cost and depreciation functions on the same set of axes to find the month at which the total cost C of owning the car surpasses its value V as it depreciates. They identify and interpret the domains on which $C > V$ and $C < V$.

Project 5.3: Linear Depreciation and the IRS

Mathematics: Linear equations, arithmetic sequences
Mathematics Learning Goals: To model linear depreciation situations using linear equations and arithmetic sequences.

Do an online search for IRS Publication 946 "How To Depreciate Property". The IRS uses a method known as "straight line depreciation". Research the depreciation conditions for business use of an automobile. How does the IRS depreciation automobiles that are used for business use? Go to an automobile sales website and select a car for business use. Determine the price

for that car. Apply the IRS depreciation equation to determine the car value for 5 years. Model the car values using an arithmetic sequence. Write the general term for the arithmetic sequence.

Project 5.4: The Physics of Driving

Mathematics: Quadratic equations, radical functions, arc length, geometry of the circle.
Mathematics Learning Goals: To use the mathematics listed to determine braking distances and to gather data from accidents scenes.

Students use formulas to determine reaction distance, braking distance, and figure out the speed a car was going based on its skid marks. The braking-distance formula is a quadratic function, with speed as the independent variable. The skid speed formula is an irrational function that has three independent variables. Students also use the geometry of the circle to compute the radius of a given yaw mark, which is a curved skid mark, and use the radius and friction factor to find the speed the car was going when it began to skid. The students then prepare a PowerPoint or poster presentation for the driver's education class in their school.

Project 5.5: Exponential Auto Depreciation.

Mathematics: Exponential equations and geometric sequences
Mathematics Learning Goals: To use the mathematics listed to model automobile depreciation.

Although the IRS uses linear depreciation for reporting purposes, in reality, items usually depreciate by a certain percentage each year. Select three different car makes and models. Do research on historical depreciation of these cars. In other words, try to find the depreciation percentage (common ratio). Write a geometric series equation for each car. Graph each function. What implications can you make from the graphs?

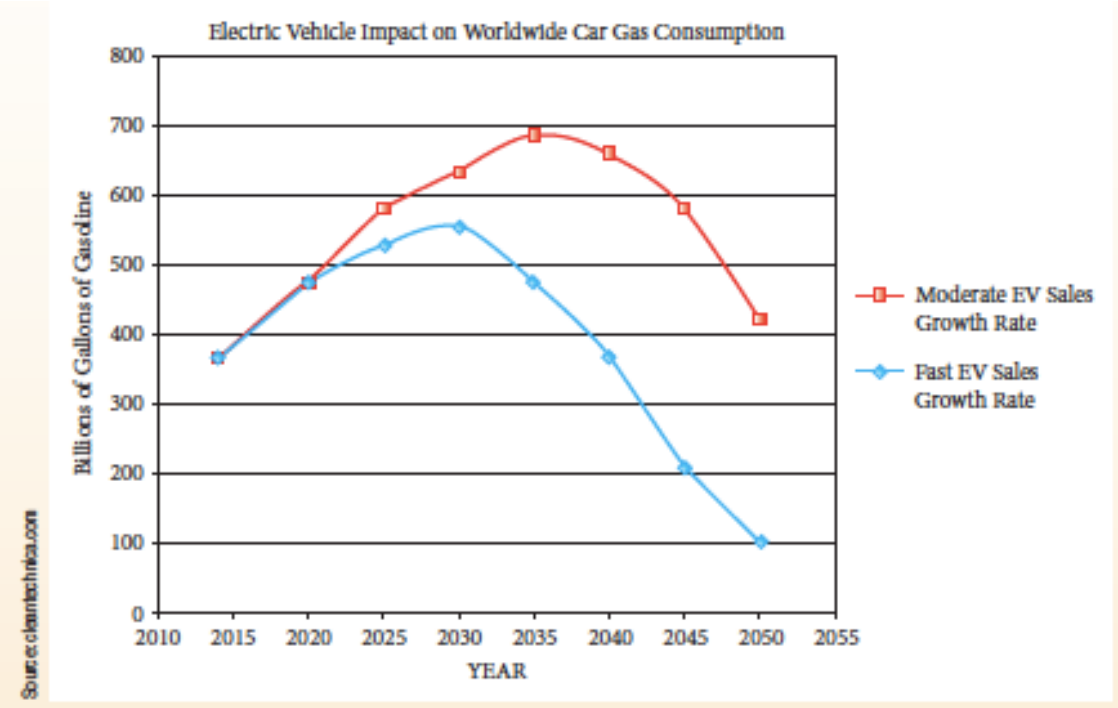
WHAT'S THE PROBLEM?

Examine the equation below used for exponential depreciation calculations. Look through your textbook and your notes to help you write a problem that could be modeled by this equation.

$$19777.84 = 24800(1 - 0.055))^4$$

YOU WRITE THE STORY!

The graph below illustrates the impact that both moderate and fast sales growth of electric vehicles around the world would be expected to have on automotive gasoline consumption. Write a short news-type article centered on this graph.



Unit 6: Consumer Credit

PROJECTS

Project 6.1: Can I Afford This Loan?

Mathematics: Exponential functions, logarithmic functions, system of exponential and linear functions, modeling, graphical interpretation
Mathematics Learning Goals: To use three modalities to determine the affordability of a loan: exponential formula evaluation, logarithmic formula evaluation, and interpreting an exponential/linear system. To use technology (graphing utility and/or spreadsheet) to make the determinations required and justify their responses.

Students are given a scenario in which a family must make a decision about the affordability of a loan based on the principal, the loan-length, the APR and the maximum affordable monthly payment the family is able to make towards loan debt reduction. Students determine the affordability of the loan in three different ways: using the monthly payment function, interpreting the graphs of the system of equations defined by the exponential monthly payment function and the linear maximum affordable monthly payment, and using the logarithmic loan length function. They are then asked to construct two spreadsheets: a monthly payment spreadsheet that charts

the monthly payment as loan length time varies from 1 to 20 years, and a loan length spreadsheet that charts time as monthly payments vary from $100 to $1000. Finally, students must write up a summary analysis for this situation explaining how the algebraic modeling by the spreadsheet formulas supports their prior work.

Project 6.2: Mathematically Modeling a Credit Card Statement

Mathematics: Algebraic modeling and spreadsheet formula creation
Mathematics Learning Goals: To algebraically model a month of activity on a person's credit card.

Students create a 21-day credit calendar that depicts algebraic representations of daily balances based upon an opening balance of Y dollars, an X-dollar purchased on the 8^{th} day, a Z dollar payment on the 13^{th} day, and a W-dollar purchased on the 20^{th} day. Using these representations from the calendar, they write algebraic expressions for the sum of the daily balances, the average daily balance, and the finance charge for this 21-day period given that the APR on this credit card is P%. Students then create a spreadsheet that models the situation described above and test their spreadsheet for a given data set.

Project 6.3: How Much Will That Student Loan Really Cost You?

Mathematics: exponential and rational functions
Mathematics Learning Goals: Students will employ the simple interest formula, the monthly payment formula and the simplified interest formula to determine the total cost of a student loan.

Students should select three different two-year colleges, four-year colleges or universities, or a career school. Go on the websites of those selected and determine the tuition cost for the upcoming school year. Assume that for each of the schools selected, the student will be financing the entire tuition costs. Go to websites such as https://studentaid.ed.gov/sa/types/loans to determine the terms and the interest rate for a 10-year Federal loan to pay off the first year of tuition. Go to websites such as http://www.finaid.org/loans/privatestudentloans.phtml to determine the terms and interest rate for a 10-year private loan. Determine how much it will cost over the life of the loan at each institution in each of the following situations: Federal subsidized loan, Federal unsubsidized loan - payment starts after graduation, Federal unsubsidized loan - interest only paid while in school then full payments made after graduation, private loan - full payments made while in school. Students are to make a presentation about their findings.

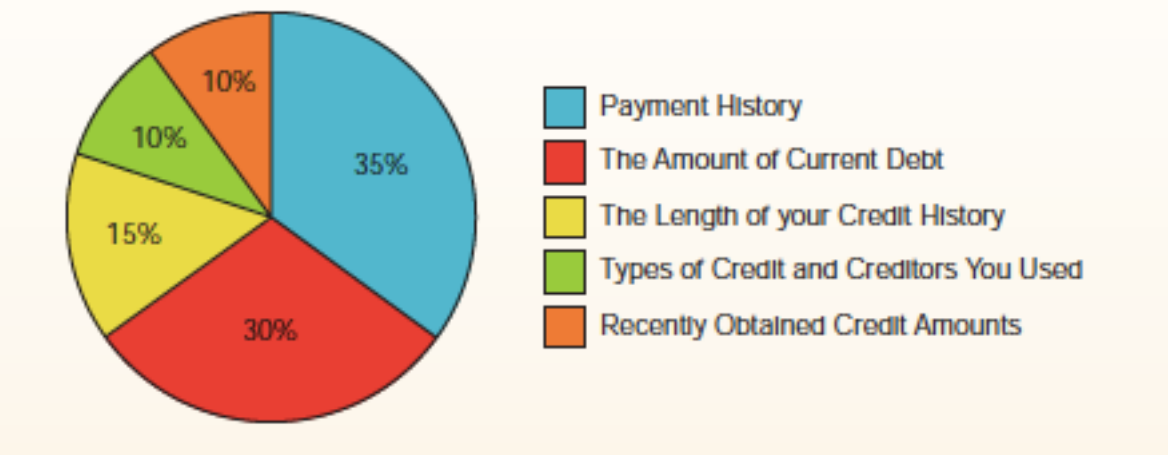
WHAT'S THE PROBLEM?

Examine the equation below used for monthly loan payment calculations. Look through your textbook and your notes to help you write a problem that could be modeled by this equation.

$$184.73 \approx \frac{18000\left(\frac{0.0429}{12}\right)\left(1+\frac{0.0429}{12}\right)^{12(10)}}{\left(1+\frac{0.0429}{12}\right)^{12(10)}-1}$$

YOU WRITE THE STORY!

FICO scores are measures of your credit risk to a potential lender. The graph below gives the weighted contribution of several factors that affect your credit rating. The Fair Isaac Corporation, the creators of the FICO score, keeps their formulas for computing the scores a secret. Write a short news-type article concerning the circle graph.



Unit 7: Independent Living

PROJECTS

Project 7.1: Areas of Irregular Plane Figures

Mathematics: Probability, ratios, random integers, graphing, random number table
Mathematics Learning Goals: To use the Monte Carlo method to find the area of any regular or irregular plane figure.

Students superimpose a grid on an irregular plane figure that is part of a landscape design. They outline the irregular figure with a rectangle and use a random number generator from a calculator, or a random number table, to generate 500 points, which they plot on their rectangular grid. As they plot each point, they note if it is inside or outside of the irregular region. They find the percent of random points that landed in the irregular region and take that percent of the area of the enclosing rectangle to approximate the area of the irregular region.

Project 7.2: Areas of Shaded Regions

Mathematics: Area formulas
Mathematics Learning Goals: To determine areas of plane figures that have sections removed from them.

As part of a unit on floor plans and interior design, students compute areas of floors to find the cost of new flooring. They also compute the cost of paint by taking the areas of the walls and subtracting window and door areas. They employ the area of a circle, square, triangle, rectangle, trapezoid, and parallelogram, and create a poster display on what a specific room cost to redo.

Project 7.3: The Apothem and the Area of a Regular Polygon

Mathematics: Inscribed circles, area of a triangle, perimeter, congruence.
Mathematics Learning Goals: To derive a formula for the area of any regular polygon.

Students use the area of a triangle to find the area of a regular polygon. They divide a regular polygon into triangles, by connecting the center to each vertex. They draw in the altitude, which is renamed the apothem, and find the area of the triangle. They discuss the congruence of the n triangles formed in the regular n-gon and multiply to find the area of the polygon. They then model this algebraically and use the commutative property of multiplication to derive the formula that the area is half the product of the apothem and the perimeter of the regular polygon.

Project 7.4: How Increased Payments Affect Mortgages

Mathematics: Rational functions
Mathematics Learning Goals: To determine the reduction in interest that extra mortgage payments result in.

Students use the monthly payment formula to compute the monthly payment for a hypothetical mortgage amount over 15 and 30 years. They compute the total payments, based on 12 monthly payments each year, and the total interest for the entire loan. They then use a mortgage calculator to assume an extra, 13th payment is made each year, so payments are made once every 4 weeks instead of once each month. They compute the interest and new total repayment period and compare the total interest to the original conventional mortgage to see the savings in total years and interest.

Project 7.5: Buying Points
Go to three lending institutions and find information about their APR for 15 and 30-year mortgages, and the cost of buying traditional and negative points. Calculate the effect that buying 1, 2 and 3 points will have on the lifetime cost of a $400,000 mortgage.

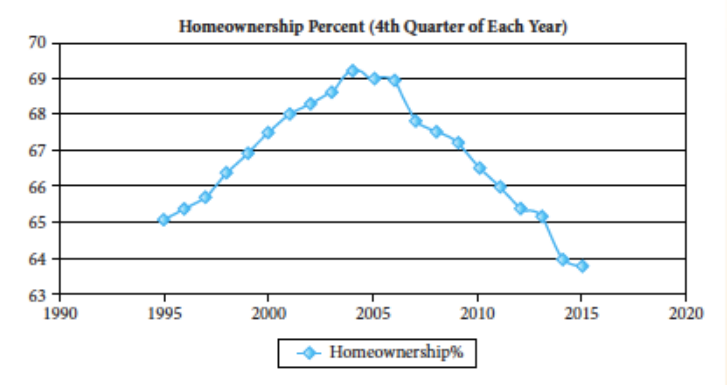
WHAT'S THE PROBLEM?

Examine the equation below used for determining the interest on a mortgage. Look through your textbook and your notes to help you write a problem that could be modeled by this equation.

$$360\left(\frac{275000\left(\frac{0.0285}{12}\right)\left(1+\frac{0.0285}{12}\right)^{360}}{\left(1+\frac{0.0285}{12}\right)^{360}-1}\right)-275000=134{,}421.81$$

YOU WRITE THE STORY!

Examine the graph below. Write a short news-type article centered on the data in the graph. The graph depicts the historical census of housing from 1995 to 2015 in the United States as reported in January 2016 by the U.S. Census Bureau.



Unit 8: Retirement and Budgeting

PROJECTS

Project 8.1: How Do Life Insurance Companies Earn a Profit?

Mathematics: Expected value, random variables, probability distributions
Mathematics Learning Goals: To use probability distributions and mortality tables to compute the profit earned on a five-year term life insurance policy.

Students use the probability inherent in mortality tables and life insurance annual premiums to compute the expected profit for a life insurance company's term policy. They create probability distributions for the random variable profit and compute expected profit by summing the products of the individual profits and probabilities for each year of the policy. They compute the minimum annual premium the company must charge to earn a profit.

Project 8.2: Planning For Retirement

Mathematics: Exponential equations, expected value, data analysis, modeling and predicting
Mathematics Learning Goals: To apply prior knowledge from the banking unit to make decisions about the feasibility of a retirement plan.

Students are given financial information about a prospective retiree and asked to act as a financial retirement planner. The prospective retiree has also supplied the planner with desired monetary goals in retirement. Based upon information about savings plans, social security benefits, pensions, and life insurance policies, and using formulas learned in this unit, the planner is to write up a financial plan for the prospective retiree that includes at least two ways of meeting the goals and has mathematical justification for the recommendations made.

Project 8.3: Cash Flow, Net Worth and Debt Reduction

Mathematics: Algebraic ratios, modeling, linear equations
Mathematics Learning Goals: To create a spreadsheet that calculates cash flow, net worth, and debt to income ratio.

Students are given a budget spreadsheet that contains the headings of income, fixed expenses, variable expenses, and non-monthly expenses. There are sub-headings under each of these listing specific categories relating to the heading. Students are given a full accounting of a person's financial status and asked to build a spreadsheet that calculates that person's cash flow. In addition, the students are given information about the person's assets and liabilities and are asked to add it to the spreadsheet and determine the net worth. Finally, based upon the calculation of the debt-to-income ratio, students are asked to develop a debt reduction plan for the individual if necessary.

Project 8.4 Budget Line Equations

Mathematics: Linear equations, domain, range, constraints, modeling,
Mathematics Learning Goals: To construct and interpret a graphical representation of a particular aspect of a budget.

A budget line graph allows the user to interpret many combinations of product usage based upon given constraints. The interpretation of the combinations allows the user to make decisions about affordability. Students are given information about a particular aspect of a personal budget. This data contains prices and budgeting constraints. Students are asked to construct a budget line equation of the form where costs are related to two budgeted items, x and y, and B is the budgeted amount. They then examine the regions above, on, and below the budget line to identify points representing affordability data. Students make recommendations for this budget item based upon the interpretation of the budget line graph.

WHAT'S THE PROBLEM 1
Examine the equation below used for determining the future value of a periodic deposit into a retirement account. Look through your textbook and your notes to help you write a problem that could be modeled by this equation.

$$\frac{1200\left(\left(1+\frac{0.01}{12}\right)^{12(30)}-1\right)}{\frac{0.01}{12}}$$

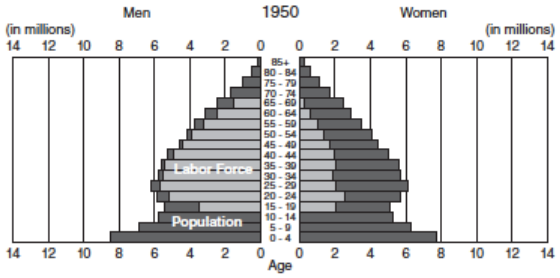
WHAT'S THE PROBLEM? 2

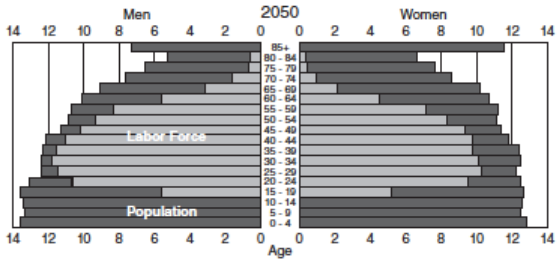
Examine the equation below used for determining the cost of an international cell phone plan, where *m*, represents the number of megabytes. Look through your textbook and your notes to help you write a problem that could be modeled by this equation.

$$36 + 0.26([375.4 - 140]m + 1) = 97.36$$

YOU WRITE THE STORY! 1

Write a short news-type article focused on the charts below from the Bureau of Labor Statistics.

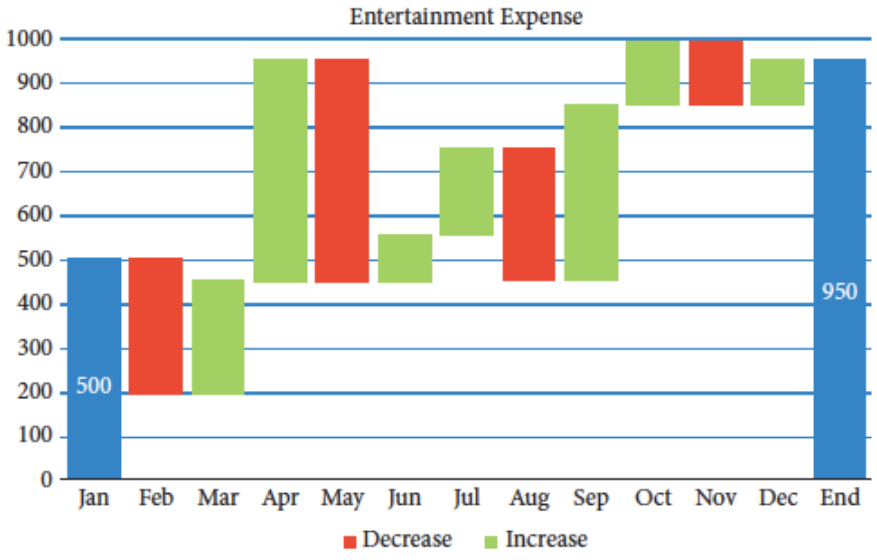




Bureau of Labor Statistics

YOU WRITE THE STORY! 2

The graph below is a type of bar graph called a "waterfall chart" for budgeted entertainment expenses from January to December. The horizontal axis represents months of the year and the vertical axis represents money budgeted in dollars. The blue bars indicate starting and ending balances in the category. The red and green bars indicate changes. Write a short news-type article centered on this graph.



MAJOR RESOURCES AND SUPPORTING MATERIALS

Geometry, Algebra 2, Precalculus, Statistics, and Probability textbooks from any publisher should be available as reference materials throughout the course.

Gerver, R. & Sgroi, R. *Financial Algebra Second Edition*. South-Western/Cengage Learning: Mason OH. Copyright 2018.

Internet Resources for Supporting Learning in the Discretionary Income Unit 1

Teach Yourself Statistics. www.stattrek.com Students can get tutorials and do computations.

Vassar Stats. www.vassarstats.net Students can perform a variety of calculations in an extensive menu of options.

Free Statistics Help Resources. http://www.statisticshelp.org This site includes online calculators, graphing, handouts and solved statistics problems.

Internet Resources for Supporting Learning in the Banking Unit 2

Federal Deposit Insurance Corporation www.fdic.gov. Students can explore how they can maximize accounts that are covered by have different types of accounts in one bank.

Loan Payment Calculator www.bankrate.com. Students can easily compare many different types of loans by changing term, rate, and principal, and they can compare the software answers to answer they get using algebraic formulas.

Internet Resources for Supporting Learning in the Investing Unit 3
The New York Stock Exchange www.nyse.nyx.com This site offers students and educators detailed and current information about the New York Stock Exchange

Yahoo! Finance finance.yahoo.com Students can access historical and daily trading information on corporate stocks.

The Stock Market Game www.stockmarketgame.org This website offers students, teachers, and parents virtual investment opportunities in an educational setting.

Thirteen edonline "Could You Start A Business?"
http://www.thirteen.org/edonline/lessons/fe_start/index.html
Teachers and students explore the financial requirements needed to start a business.

Investopedia www.investopedia.com This is a comprehensive investment website for students and adults. It covers topics on investment, the stock markets, personal finance, trading, and much more. It is an excellent source of investing information for many of the student projects.

Internet Resources for Supporting Learning in the Employment/Income Taxes Unit 4

Internal Revenue Service www.irs.gov. Besides getting all the forms and instructions from this website, students can explore Publication 17: Your Federal Income Tax to find out all about tax deductions and tax credits.

Social Security Administration www.ssa.gov. Students can use this site to explore social security credits, benefits, and application procedures.

Department of Labor www.dol.gov. This site allows students to look up labor laws that affect all of them. Students should also go to their specific state's Department of labor website for state-specific regulations.

Internet Resources for Supporting Learning in the Automobile Ownership Unit 5

Kelly Blue Book www.kbb.com This website gives students access to real-time data for new and used car sales.

Gas Buddy www.gasbuddy.com Here, students are able to get up to date information about gas prices throughout the country.

Teen Driver Source www.teendriversource.org/teen This website is sponsored by the Children's Hospital of Philadelphia. It offers a great deal of research and statistical information that can assist teens to become better drivers.

Motor Vehicle Crash Investigation and Reconstruction
www.Crashforensics.com/papers.cfm?PaperID=37
This website is used as a student reading assignment in the section on accident reconstruction.

Insurance Institute for Highway Safety www.iihs.org
Here, students are offered information and statistics on a variety of highway safety topics.

Internet Resources For Supporting Learning in the Consumer Credit Unit 6

The Federal Reserve http://www.federalreserve.gov/consumerinfo/fivetips_creditcard.htm
This easy to read website offers students suggestions for managing a credit card account.

Index Credit Cards http://www.indexcreditcards.com/creditcardlist.html
This website offers students detailed information about all credit cards that are available. Here they can compare the terms of the credit card agreements and find information specific to different types of credit cards.

Teens' Guide To Spending www.teensguidetomoney.com/spending/
The "Teens Guide To Money" website has 7 links to spending and credit topics of interest to teenagers.

MSN Money
http://money.msn.com/credit-rating/article.aspx?post=45196da9-83b9-4229-8435-14415c64fe16
This is a teen's guide to understanding credit scores.

The College Board
https://bigfuture.collegeboard.org/pay-for-college/loans/8-tips-for-taking-out-student-loans
The College Board offers students tips for taking out student loans and paying for college.

Internet Resources For Supporting Learning in the Independent Living Unit 7

Khan Academy
http://www.khanacademy.org/economics-finance-domain/core-finance/housing/renting-v-buying/v/renting-vs--buying-a-home
This educational video walks students through the mathematics of comparing home rental vs a home purchase.

New York Times
http://www.nytimes.com/interactive/business/buy-rent-calculator.html
This is an interactive rent vs buy calculator that gives monetary information in a variety of formats.

Mortgage Calculator http://www.mortgagecalculator.org
This is one of the many mortgage calculator sites available online.

Mortgage Points Calculator
http://www.bankrate.com/calculators/mortgages/mortgage-loan-points-calculator.aspx
This is one of the many mortgage point calculator sites available online.

Internet Resources for Supporting Learning in the Budgeting/Retirement Planning Unit 8

Mapping Your Future www.mappingyourfuture.org. Students can get budget tips and then plan a budget by entering their expenses into a software template.

Practical Money Skills www.practicalmoneyskills.com. This site gives students access to a budget worksheet and budget calculators.

Budget Calculator www.bankrate.com. This site offers a comprehensive budget calculator students can use to examine increases and decreases in expenses and how they affect a budget. The speed of the software's recalculation allows students to repeatedly explore the affect of changing expenses.

INSTRUCTIONAL METHODS AND STRATEGIES

The instructional strategies used throughout this course are varied, targeted, and rooted in standards. The methods used in this course are field tested and research-based. The practices referenced in this section serve to build mathematical confidence, interest and strength.

The Advanced Algebra with Financial Applications program's instructional strategies cover these basic umbrellas:

- Motivational Unit Openers
- Essential Questions
- Reading
- Discussion/interaction
- Presentation of model problems
- Extensions and problem solving
- Differentiation of instruction
- Experiential learning
- Use of technology

The **motivational unit openers** are real-life problems that need to be solved mathematically. Students realize that they "need to know" this material, as they will be encountering financial matters every days of their adult lives. Financial situations are inherently natural motivators. Since all of the problems in the course are real-world applications, lessons must integrate **reading and discussion** on a daily basis. An **essential question**, written on the board each day, serves as a focal point as algebraic symbols are used to represent the situation. These applications are all embedded in prose, so every new topic begins with a reading passage that acts as a springboard to a full-class discussion. This lively interactive feature of every lesson sets a constructive, motivating stage for the mathematics that follows.

The direct instruction/lecture component is highlighted by the investigation of **model problems** on each skill covered. After each model problem, students look for structure and regularity and try to apply it in a situation rooted in the model problem just completed. This gives the students a chance to see if they understood the new concept before moving on to a deeper problem for which the previous problem was an entry condition. Students are then asked to extend their understanding by looking for patterns and **extending** previously-used strategies. The applications at the end of each section give students a chance to practice as part of their classwork and homework. The program spirals previously-learned material on a daily basis. The sequential nature of the introduction of each new skill, followed by immediate practice, allows students to monitor their progress often. Class notes include vocabulary and financial explanatory material as well as mathematical procedures.

The model problems and applications generally graduate in difficulty level, allowing the teacher to **differentiate instruction**. Since abstract reasoning can be difficult for many students, the instructions are graduated so students can grasp the higher level skills by meeting them step-by-step. This strategy allows student and teacher to identify the exact juncture at which the student is having difficulty. This makes diagnostics and intervention more pointed.

The course offers a great opportunity for **experiential learning**. Projects require the students to get out in the field and meet with brokers, bankers, local businesses, etc. Guest speakers at several junctures bring the outside world right into the classroom. Students act as moderators and compile questions for the guest speaker. For some projects, data is gathered and statistically analyzed. Students present their work to the class, and they field questions and comments from their classmates.

Technology plays a key role in the development of Advanced Algebra with Financial Applications topics. The graphing calculator and online graphing sites such as DESMOZ are extensively used daily tools. Spreadsheets appear in every unit so students can model situations using algebra and technology.

(D)
METHODS OF EVALUATING STUDENT OUTCOMES

A variety of formative and summative assessment methods are used throughout Advanced Algebra with Financial Applications in order to assess student learning. The assessments are aligned with the course purpose and the instructional strategies used for the development of mathematically proficient students. In the activities listed below, students are offered assessment opportunities to address mathematics as a sense-making tool, problem solve, reason, construct arguments, offer mathematics-justified critiques of arguments, model, use appropriate tools, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning. The assessment grading percentages contributing to the student's quarter course grade are offered in parentheses next to the assessment name.

FORMATIVE ASSESSMENTS (30%)

CLASS PARTICIPATION (15%)

- **Do Now Activities** are assessments that can be used as a vehicle for the teacher to determine whether students have acquired skills, strategies, and content necessary for subsequent work in a topic. This diagnostic feature allows the teacher to adjust the lesson accordingly, if entry conditions are not fully met.

- **Warm-Ups** are short, formative assessment questions that determine whether or not students have met prerequisite material necessary for success in the upcoming lesson.

- **Check Your Understanding** problems are offered to students immediately after the teacher has introduced a new concept or procedure. These problems offer students and teacher alike an immediate assessment opportunity that is confined to the single new skill just addressed. The teacher can adjust the lesson to follow based upon review of these problems.

- **Extend Your Understanding** problems are more advanced problems that use the concepts and procedures just learned and take them to another level. These can be

offered to all students or differentiated for selected students depending on the nature of the problems.

- **Ticket to Leave Activities** are ungraded activities that offer the teacher an opportunity to determine the level of understanding students acquired on the skills, strategies, and content of the day's lesson. These activities can be used by the teacher to adjust the following day's lesson.

- **Direct and Indirect Teacher Questions** are immediate formative methods of assessing students' understanding. In-class discussion is a critical part of Advanced Algebra with Financial Applications. The teacher should initiate discussion through focused questioning.

- Through the **Exploration of Essential Questions** (one per lesson), the teacher assesses student understanding both pre-instruction and post-instruction. The essential question is offered to the students at the beginning of the first lesson on the topic and a discussion ensures. That same essential question is revisited during the instruction and/or post-instruction to assess student growth and learning.

- Reading and writing are an essential part of Advanced Algebra with Financial Applications. Teachers will use **written and oral response to reading** (from the textbook, newspapers, magazines, Internet, brochures, laws, etc.) as a way of assessing understanding. Some writing activities will offer students an opportunity to interpret data that is displayed in a pictorial representation. Based upon the data, they are asked to write a short, newspaper-type story centered on the graph. There is one such activity for each chapter.

HOMEWORK (15%)

- **Homework Assignments** are a daily evaluation and reflection device for both student and teacher. The level of proficiency with the homework questions should allow the teacher the opportunity to adjust the lesson as needed. The homework acts as a barometer for students, so they can formulate questions, and attempt problems on their own.

SUMMATIVE ASSESSMENTS (70%)

- **Lesson-Opener Quizzes** are short, graded, cumulative assessments that can test for prerequisite skills and/or mastery of recently taught material. These assessments are averaged and count as one full-period exam grade.

- **Full-Period Exams** are graded summative assessments that test student acquisition of skills, strategies, and content.

- **Experiential Learning** activities are project-based assessment tools that are offered to students as long-term assignments. Students are asked to do research in a variety of

forms and formats in order to accomplish a task that is related to the skills, strategies, and content covered in the chapter. Their projects can be submitted in print, electronic, or presentation format. Precision and accuracy will be scrutinized during their presentations as well as the ability to use mathematical tools appropriately and strategically. Each project is valued as a single full-period exam grade.

(E)
QUALIFICATIONS OF THE TEACHER

Teachers of Advanced Algebra With Financial Applications need not have a background in finance or economics. This course contains topics in Algebra 2, Precalculus, Geometry, Trigonometry, Statistics, and Probability. The teacher who is assigned to this course should be qualified to teach these topics.

(F)
NUMBER OF CREDITS REQUESTED

Advanced Algebra with Financial Applications is a full-year course that earns one credit. It has Algebra 1 as a prerequisite.